AstroBEAR 2.0
AMR Multi-physics for Astrophysics
and
HEDP Astrophysics

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SIMULATING MAGNETOHYDRODYNAMICAL FLOW WITH CONSTRAINED TRANSPORT AND ADAPTIVE MESH REFINEMENT: ALGORITHMS AND TESTS OF THE AstroBEAR CODE

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- Solve hyperbolic PDE with elliptic constraints: **MHD**
- Source terms for energy loss/gain, **ionization dynamics, Real EOS**
- Operator splitting: **gravity, heat conduction** (*HYPRE*)
- **Sink Particle**/ Particle Dynamics

$$\frac{\partial}{\partial t}\begin{bmatrix}\rho\\ \rho v_x\\ \rho v_y\\ \rho v_z\\ \mathcal{E}\\ B_x\\ B_y\\ B_z\end{bmatrix}+\frac{\partial}{\partial x}\begin{bmatrix}\rho v_x\\ \rho v_x^2+P+B^2/2-B_x^2\\ \rho v_y v_x\\ \rho v_z v_x\\ (\mathcal{E}+P+B^2/2)v_x-B_x(\boldsymbol{B}\cdot v)\\ 0\\ -E_z\\ E_y\end{bmatrix}+$$

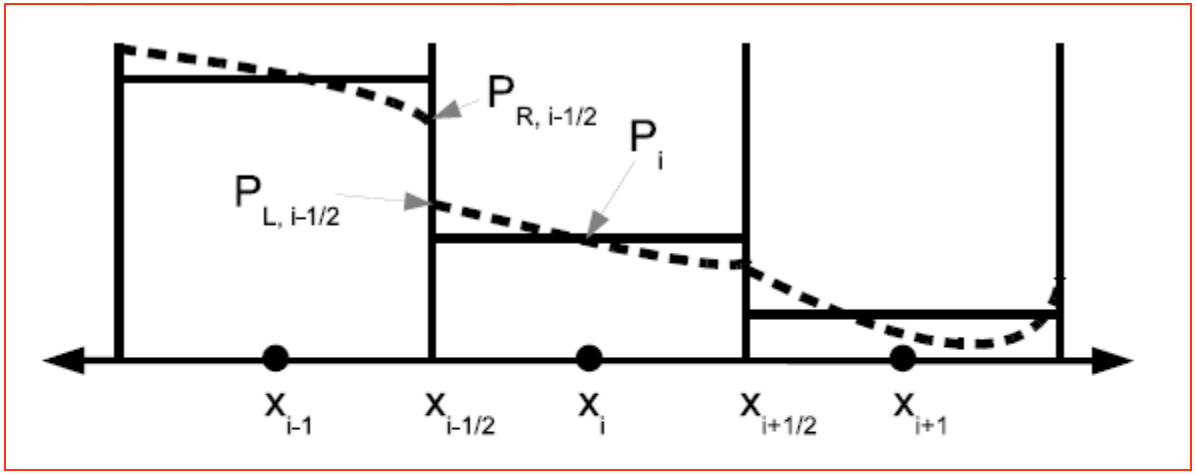
$$\frac{\partial}{\partial y}\begin{bmatrix}\rho v_y\\ \rho v_x v_y\\ \rho v_y^2+P+B^2/2-B_y^2\\ \rho v_z v_y\\ (\mathcal{E}+P+B^2/2)v_y-B_y(\boldsymbol{B}\cdot v)\\ E_z\\ 0\\ -E_x\end{bmatrix}+\frac{\partial}{\partial z}\begin{bmatrix}\rho v_z\\ \rho v_x v_z\\ \rho v_y v_z\\ \rho v_z^2+P+B^2/2-B_z^2\\ (\mathcal{E}+P+B^2/2)v_y-B_z(\boldsymbol{B}\cdot v)\\ -E_y\\ E_x\\ 0\end{bmatrix}=\boldsymbol{S}$$

$$\nabla\cdot\boldsymbol{B}=0.$$

AstroBEAR Basics
Reconstruction

$$Q^{t+\Delta t}_{i,j,k} = Q^{t}_{i,j,k} + \frac{\Delta t}{\Delta x}\left(\tilde{F}^{n}_{x,i-1/2,j,k} - \tilde{F}^{n}_{x,i+1/2,j,k}\right) + \frac{\Delta t}{\Delta y}\left(\tilde{F}^{n}_{y,i,j-1/2,k} - \tilde{F}^{n}_{y,i,j+1/2,k}\right) + \frac{\Delta t}{\Delta z}\left(\tilde{F}^{n}_{z,i,j,k-1/2} - \tilde{F}^{n}_{z,i,j,k+1/2}\right).$$

where $\tilde{F}^{n}_{x}$, $\tilde{F}^{n}_{x}$, and $\tilde{F}^{n}_{x}$ are suitably accurate numerical approximations to the inter-cell flux, spatially averaged over the inter-cell area and temporally averaged from t to $t + \Delta t$.



AstroBEAR Basics
Riemann Solver and Time Stepping

$$\tilde{F}_{x,i-1/2|} = \frac{1}{2}\left(F_x(Q_{L,i-1/2}) + F_x(Q_{R,i-1/2})\right) - \frac{1}{2}\sum_{m=1}^{8} L_{m,i-1/2}\left(a_{m,i}\left[Q_{R,i-1/2} - Q_{L,i-1/2}\right]\right) R_{m,i-1/2}.$$

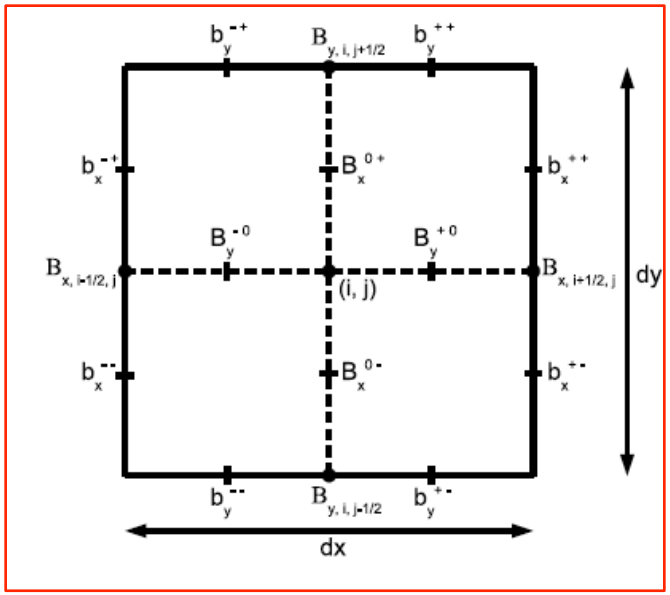
The MUSCL-Hancock predictor-corrector temporal discretization achieves second order accuracy by advancing the grid-face interpolated states by a half-time-step using a one dimensional predictor scheme. The predictor scheme is carried forward according to:

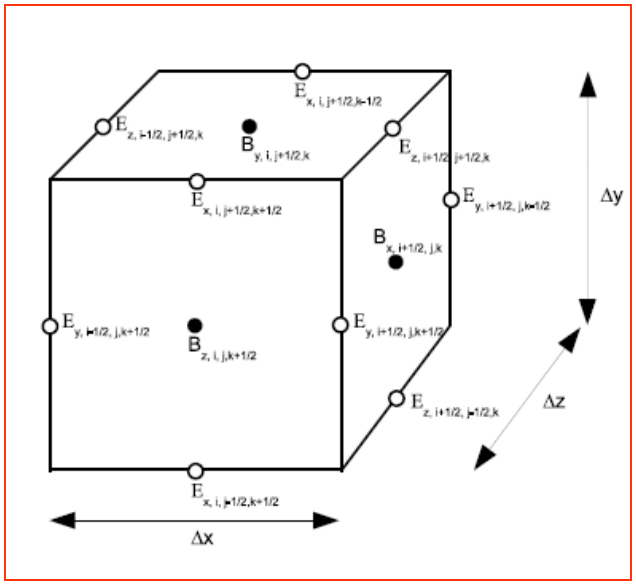
$$\begin{aligned} Q^{t+\Delta t/2}_{L,i+1/2} &= Q^{t}_{L,i+1/2} + \frac{\Delta t}{2\Delta x}\left(F_x(Q^{t}_{R,i-1}) - F_x(Q^{t}_{L,i+1})\right) + \frac{\Delta t}{2} S(Q_i) \\ Q^{t+\Delta t/2}_{R,i-1/2} &= Q^{t}_{R,i-1/2} + \frac{\Delta t}{2\Delta x}\left(F_x(Q^{t}_{R,i-1}) - F_x(Q^{t}_{L,i+1})\right) + \frac{\Delta t}{2} S(Q_i). \end{aligned} \tag{19}$$

Note that the predictor step uses the cell centered, volume average fluxes. The Riemann problem at the cell interfaces is not solved and the upwinded flux at the cell-faces are not needed for this step.

AMR MHD: Div B = 0

- Hydro: Need conservative prolongation/ restriction operators.
- MHD: Maintain solenoidal condition. Need divergence free operators on “staggered mesh”.





$$(\nabla \cdot B)^{t+dt}_{i,j,k} = (\nabla \cdot B)^{t}_{i,j,k}$$

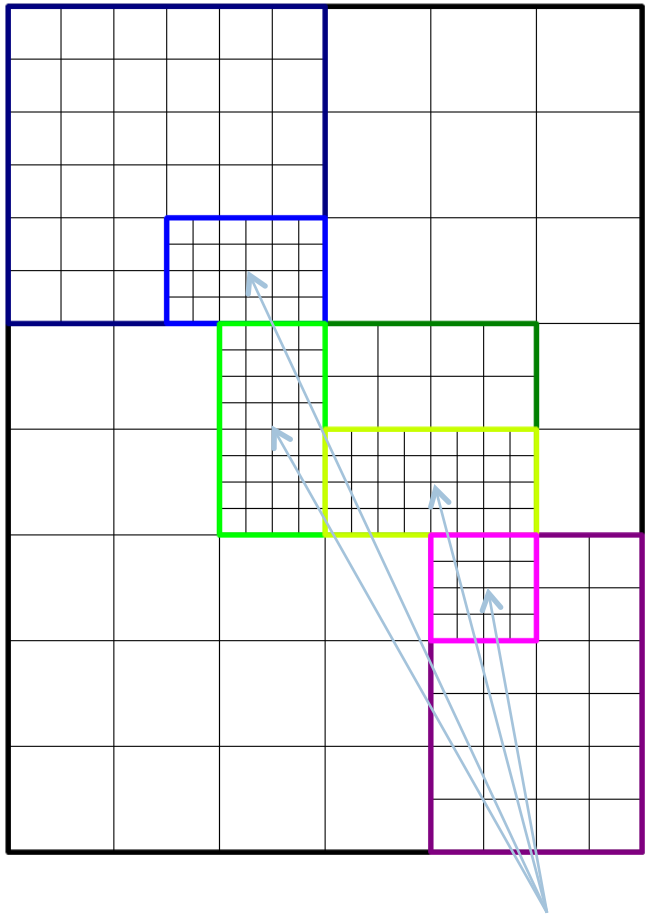
AstroBEAR 1.0 MHD-AMR Code

AstroBEAR strategy: Use a variety of methods

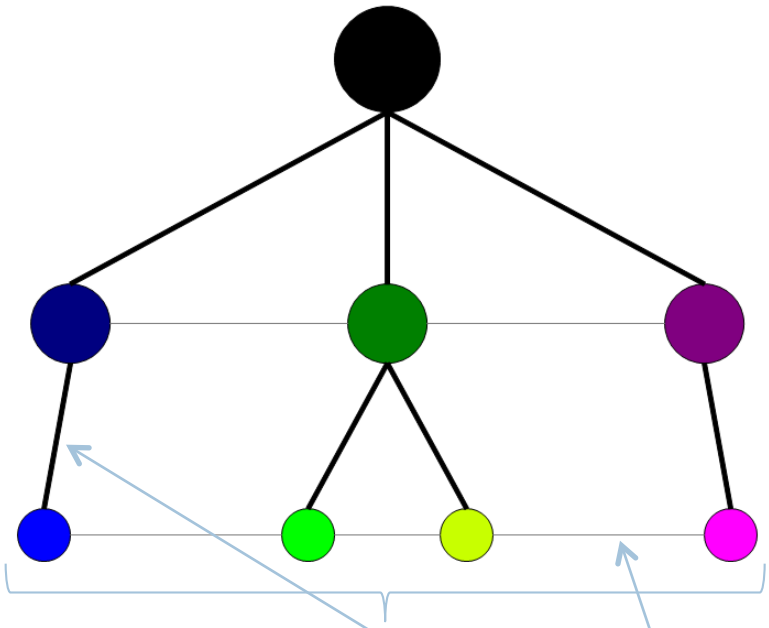
- Reconstruction: Muscl, PPM, PPH
- Riemann Solvers: Roe, HLLD, Marquina
- Integration Schemes: RK, Muscl-Hancock

MHD CT: Ryu et al (95),
Balsara & Spicer (99)

Mesh



Tree

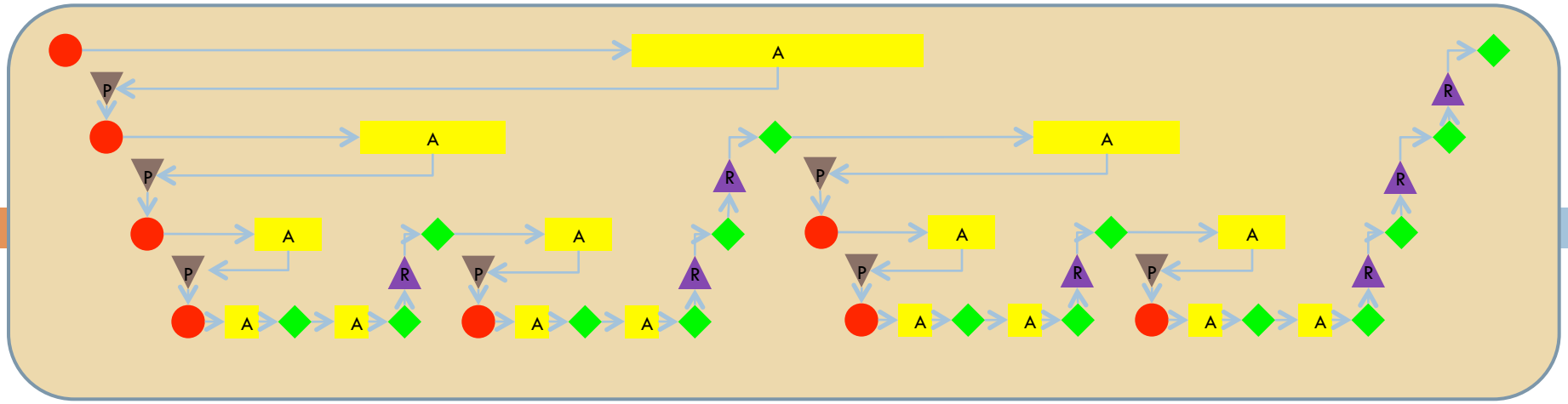


Level 2 nodes are children of level 1 nodes

The AMR tree contains all of the **parent-child** relationships between grids on different levels as well as **neighbor** relationships between grids on the same level.

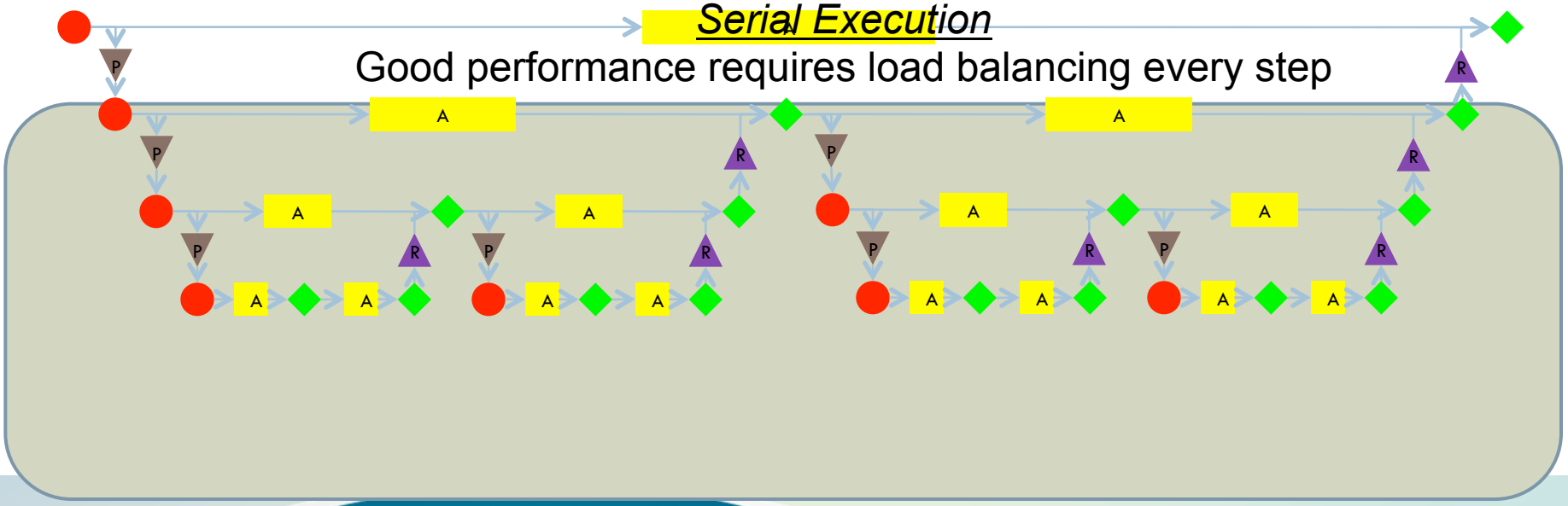
Level 2 grids nested within parent level 1 grids

AMR Parallel Performance
AstroBEAR 2.0 Distributed Tree



Serial Execution

Good performance requires load balancing every step



Threaded Execution

Good performance requires global load balancing across AMR levels

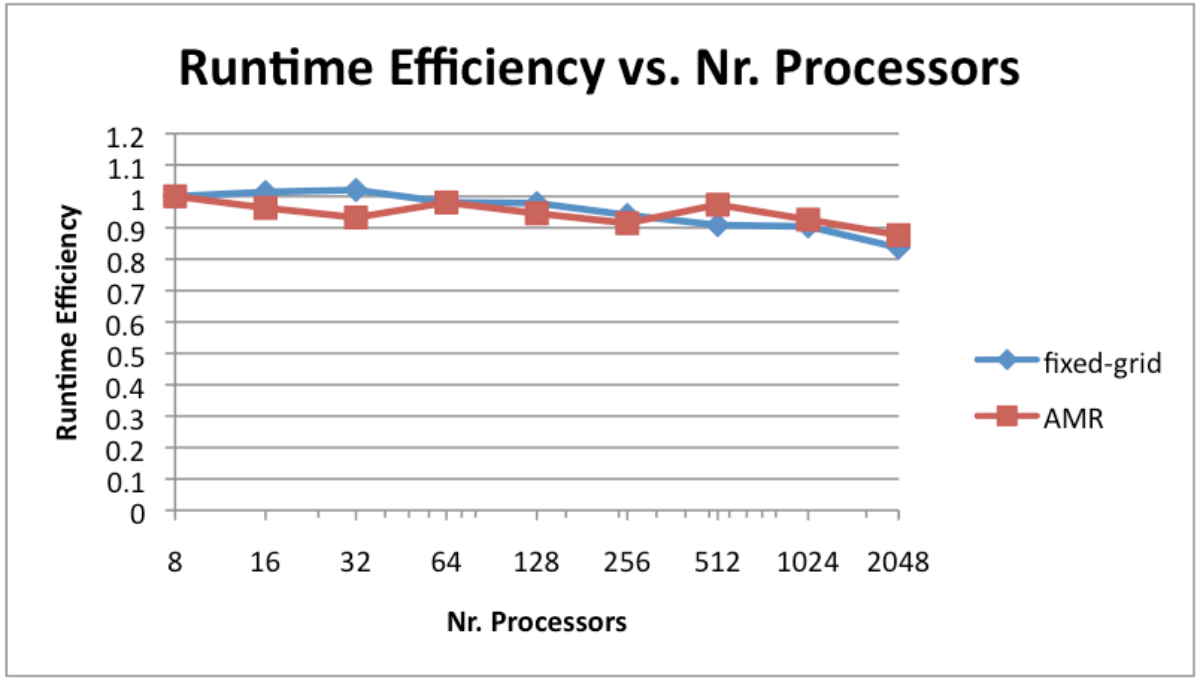
AMR Parallel Performance

AstroBEAR 2.0 Level Threading

AB 2.0: Parallel AMR Performance

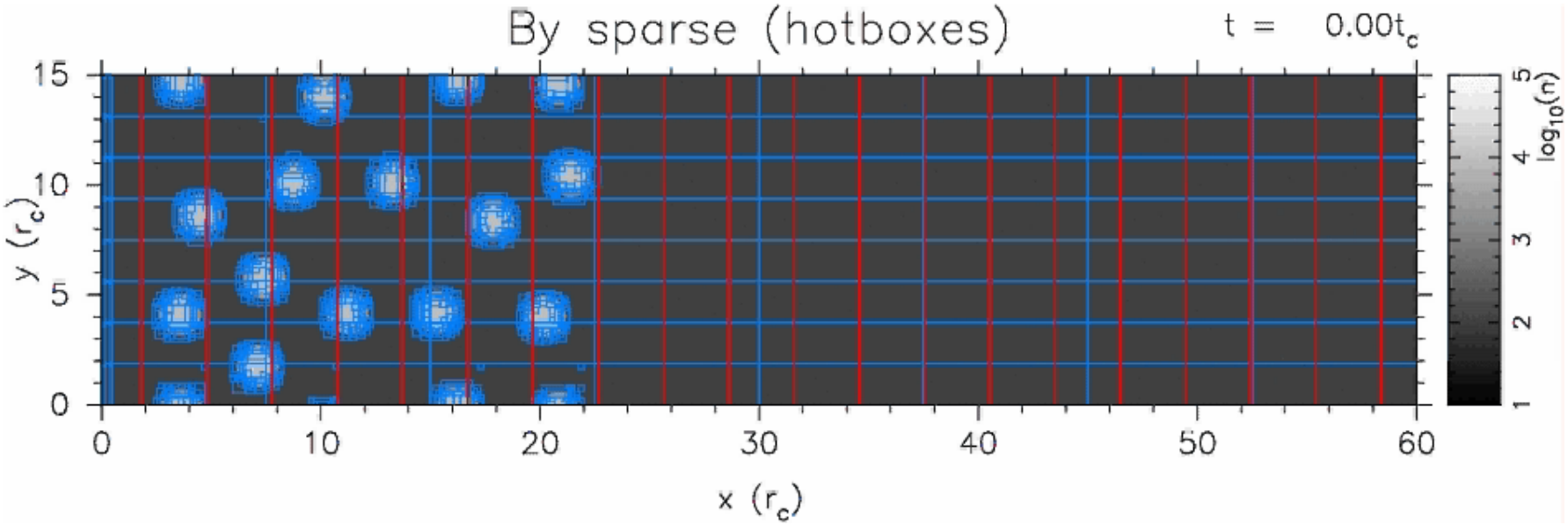
Carroll, Shroyer, Frank & Ding 2011

- Scrambler: Load balance AMR grid hierarchy



Shocked Radiative MHD Heterogeneous Flows
(Frank et al 2011)

- Low Clump Filling Fraction

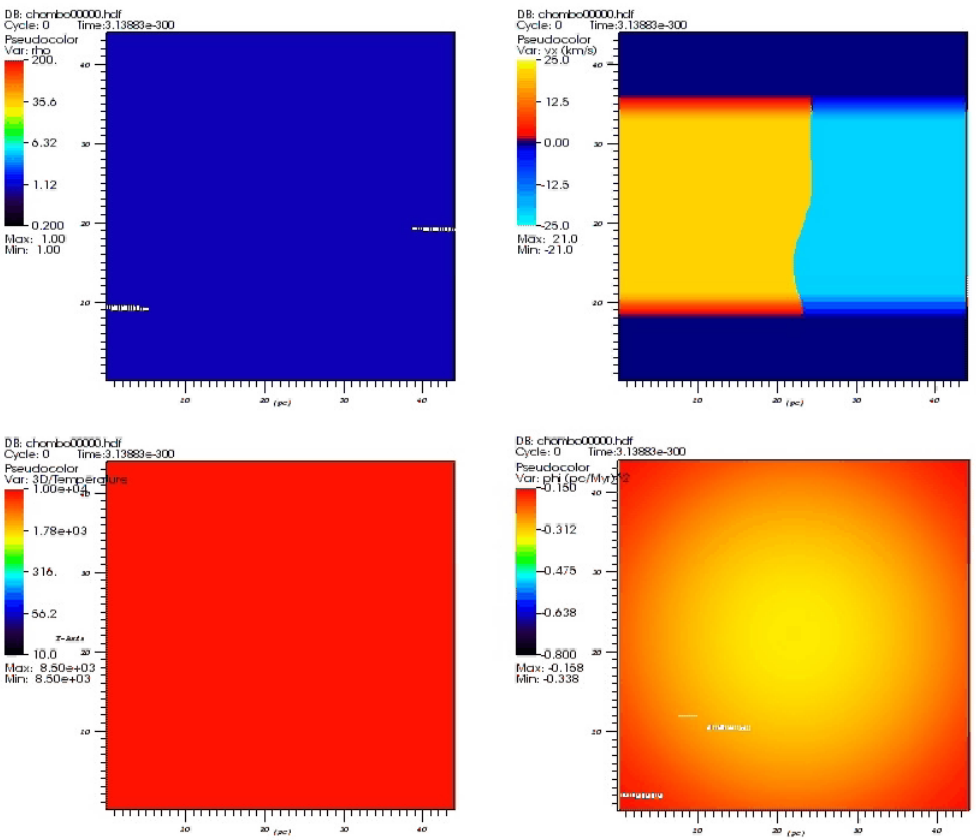


Star Formation Studies w AstroBEAR 2.0 Carroll et al 2011

<u>Molecular Clouds:</u>
Origins as colliding flows

Hartmann et al 2002

Heitsch et al 2008

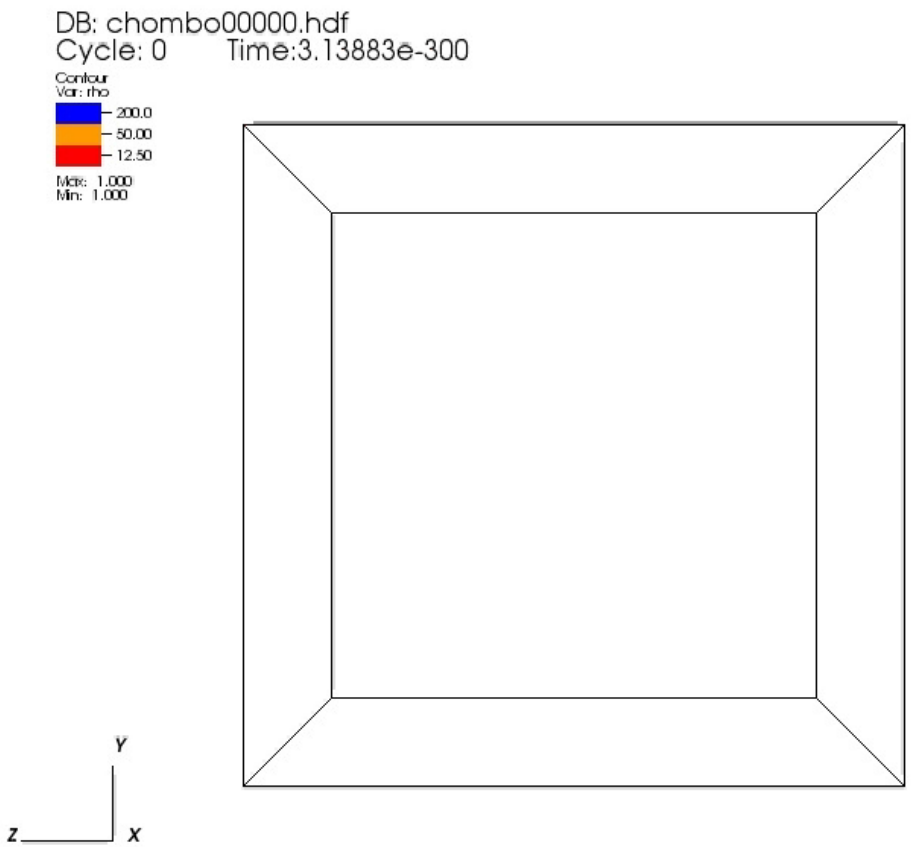


Temp

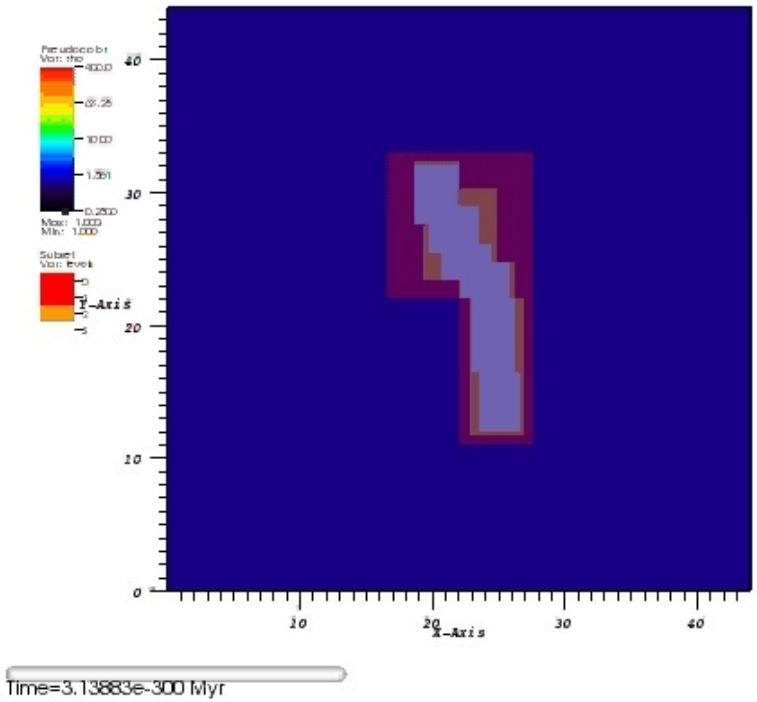
Potential

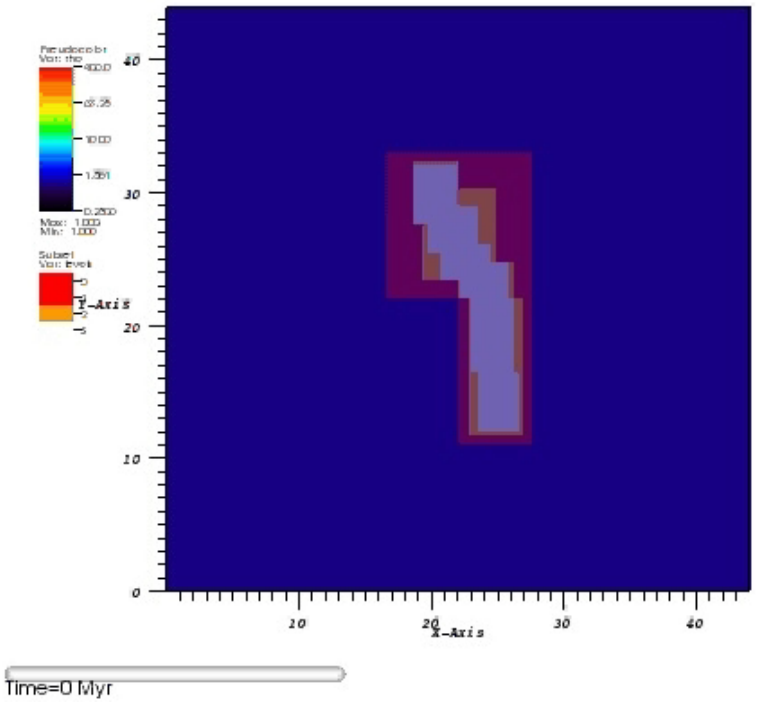
Star Formation Studies w/ AstroBEAR 2.0
Carroll et al 2011

Iso-surfaces
Of Density



Star Formation Studies w/ AstroBEAR 2.0





2 Runs: Different AMR Refinement Criterion.
Maps of Density

Clumpy Jet Models: Yirak et al 2011
Synthetic Observations Green Hα Red [SII]

0 degrees

20 degrees

Magnetic Tower Simulations

Huarte-Espinoza et al 2011

The magnetic vector potential is given by

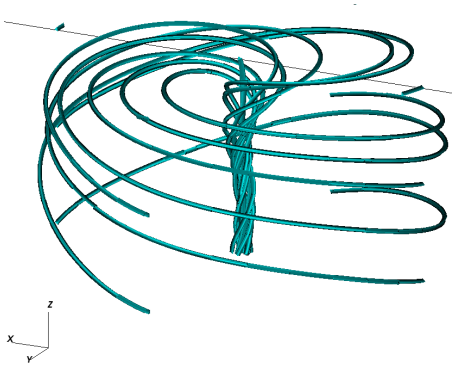
$$\mathbf{A}(r,z) = \begin{cases} \frac{r}{4}(cos(2\,r)+1)(cos(2\,z)+1)\hat{\phi} + \frac{\alpha}{8}(cos(2\,r)+1)(cos(2\,z)+1)\hat{k}, & \text{for } r,z < r_t; \\ 0, & \text{for } r,z \geq r_t, \end{cases} \tag{2}$$

in cylindrical coordinates, where $r_t = \pi/2$ is the radius of the magnetic injection region. The parameter α is an integer with units of length. Inside the column, $\mathbf{A}$ can also be written as

$$\mathbf{A}(r,z) = rcos^2(r)cos^2(z)\hat{\phi} + \frac{\alpha}{2}cos^2(r)cos^2(z)\hat{k}. \tag{3}$$

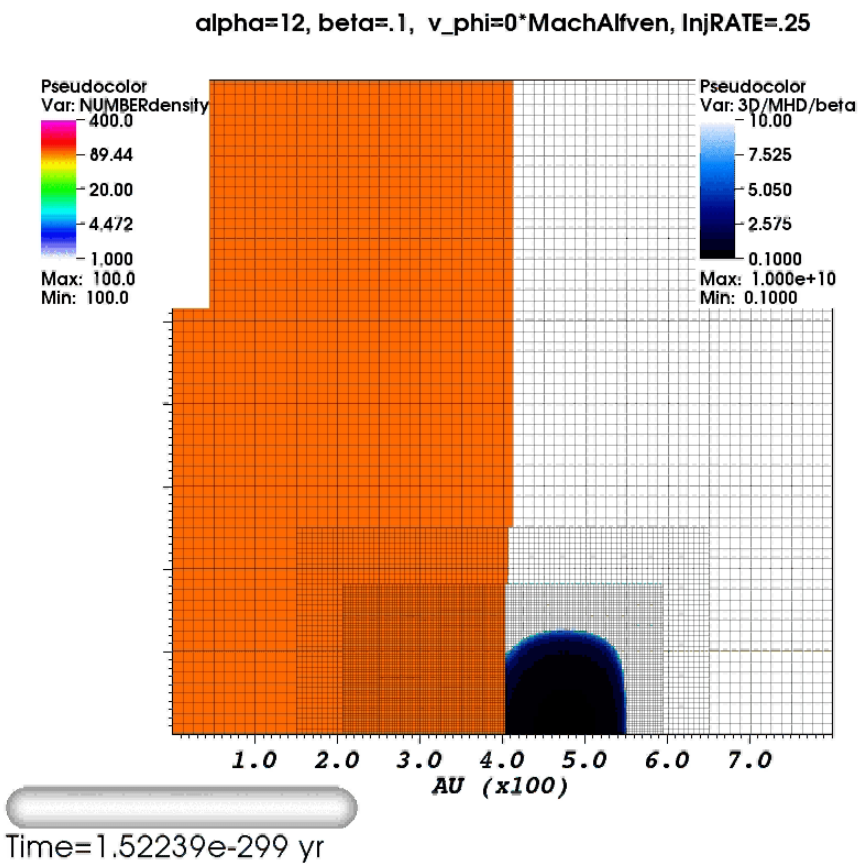
From $\mathbf{B} = \nabla \times \mathbf{A}$,

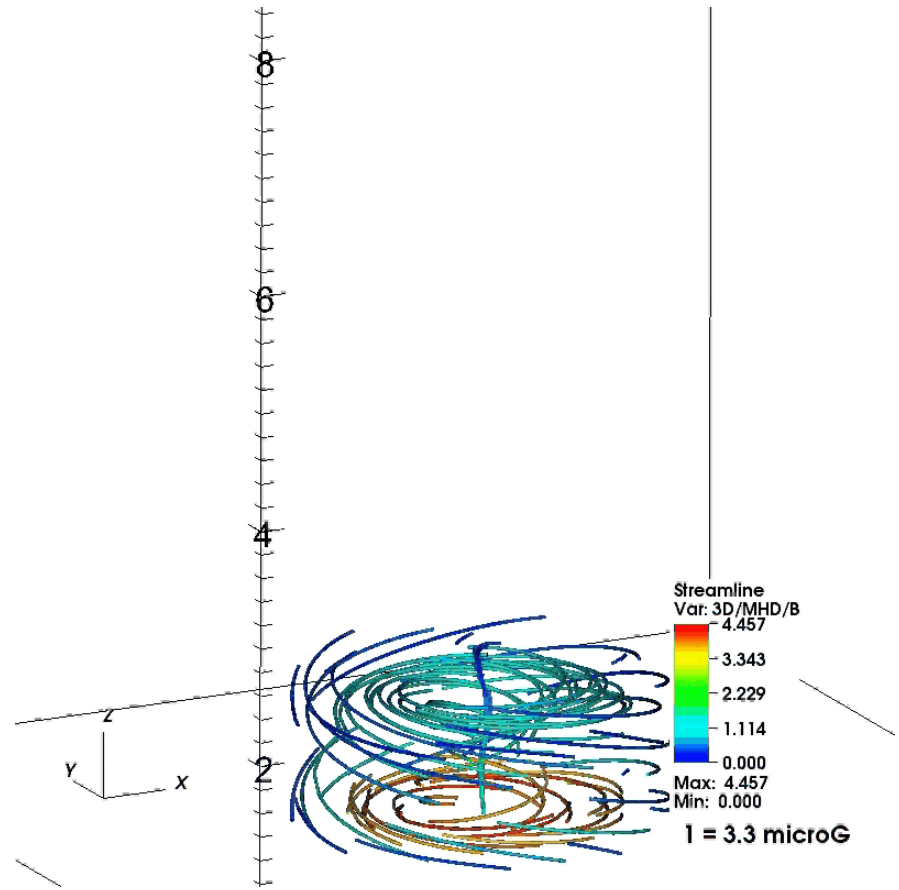
$$\begin{aligned} B_r &= -\tfrac{\partial}{\partial z}(A_\phi) = 2rcos^2(r)cos(z)sin(z), \\ B_\phi &= -\tfrac{\partial}{\partial r}(A_z) = \alpha cos^2(z)cos(r)sin(r), \\ B_z &= \tfrac{1}{r}\tfrac{\partial}{\partial r}(rA_\phi) = 2cos^2(z)(cos^2(r) - rcos(r)sin(r)). \end{aligned}$$



Magnetic Tower Simulations

Huarte-Espinoza et al 2011





Microphysical Resistivity in AstroBEAR

In AstroBEAR, we solve the magnetic diffusion equation
in addition to the ideal MHD equations using operator splitting method.

The equation $\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times (\eta \nabla \times \mathbf{B})$

The diffusivity η can be derived from the Spitzer resistivity for fully ionized plasma. In Gauss unit, it is:

$$\eta = 8.2243 \times 10^5 \times Z_{eff} F(Z_{eff}) \ln \Lambda \, T^{-3/2} \quad (cm^2 / s)$$

where T is the electron temperature in eV, Z_{eff} is the effective ion charge. The Coulomb logarithm is given by:

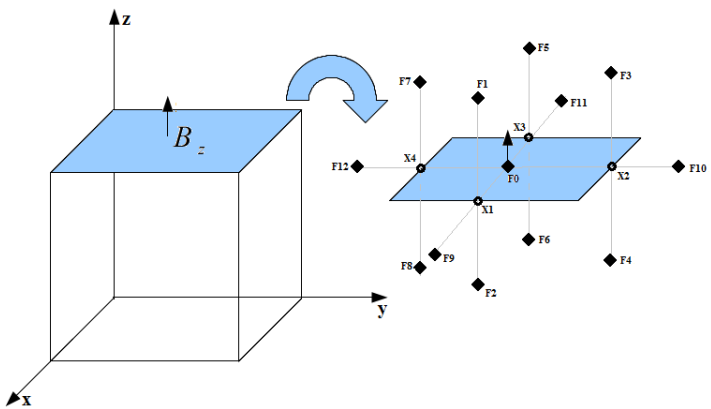
$$\ln \Lambda = \begin{cases} 1.5 \ln T - 0.5 \ln n - \ln Z_{eff} + 16.09, & (T < 10 Z_{eff}^2 \ ev) \\ \\ \ln T - 0.5 \ln n + 17.09, & (T > 10 Z_{eff}^2 \ ev) \end{cases}$$

Here, T is the same as before, n is the electron number density in the units of per cubic centimeter.

The "F" function in the diffusivity definition can be written as:

$$F(Z_{eff}) = \frac{1 + 1.198 Z_{eff} + 0.222 Z_{eff}^2}{1 + 2.966 Z_{eff} + 0.753 Z_{eff}^2}$$

Resistivity Implementation (Li)



Compute the induced "emf" along the four edges: x1, x2, x3, x4:

$$\Theta = -\eta \nabla \times \mathbf{B}$$

"emf" on each edge is obtained by computing the curl of magnetic field at that edge, multiplied by the local diffusivity, which is obtained from the edge centered temperature and electron number density. For instance, to calculate the "emf" at edge x4, we need to first find the averaged temperature and electron number density at x4 to calculate a local diffusivity, then calculate the edge centered curl of magnetic field using the available field components at face centers F0, F7, F8, F12.

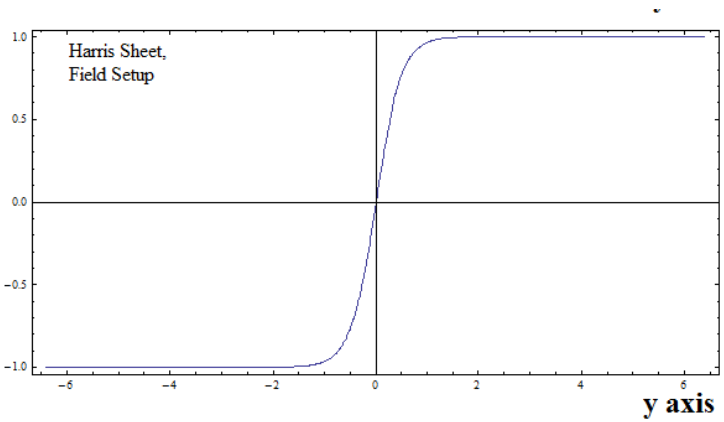
Once we find the "emf" along the four edges, we can calculate the field change by:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \Theta$$

To avoid field divergence when doing AMR, it is important to keep track of and store the edge centered "emf".

The Current Sheet Outflows Generated by Local Magnetic Reconnection

Initial Field



The Current Sheet Reconnection Outflow Test

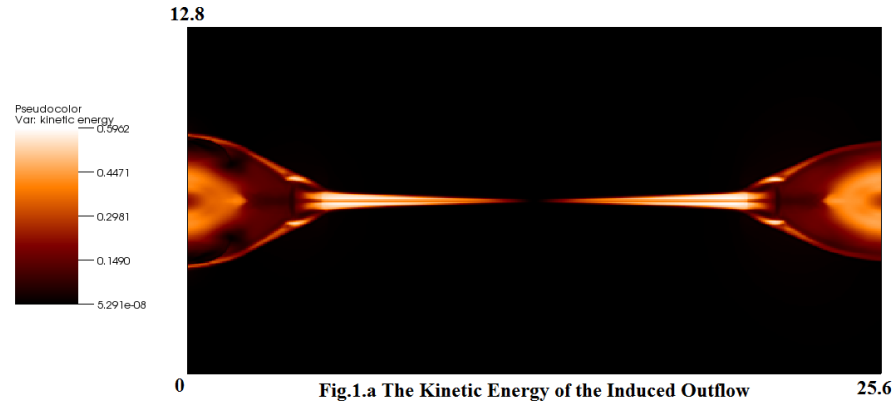


Fig.1.a The Kinetic Energy of the Induced Outflow

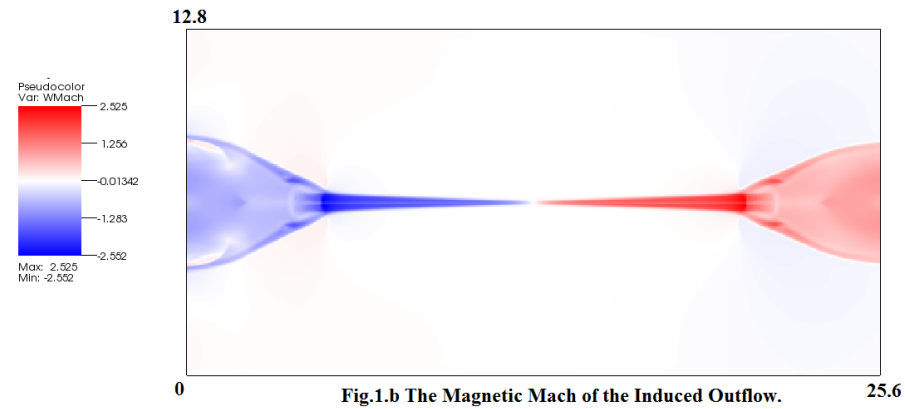


Fig.1.b The Magnetic Mach of the Induced Outflow.
The Signs Indicate the Flow Directions.

The initial setup is demonstrated by the diagram on the left.
The field points to opposite directions on the upper and lower half planes.
The temperature is initially uniform, the total pressure is in equilibrium at the beginning.
We setup the computational diffusivity so that the small area surrounding the origin has slightly increased diffusivity.

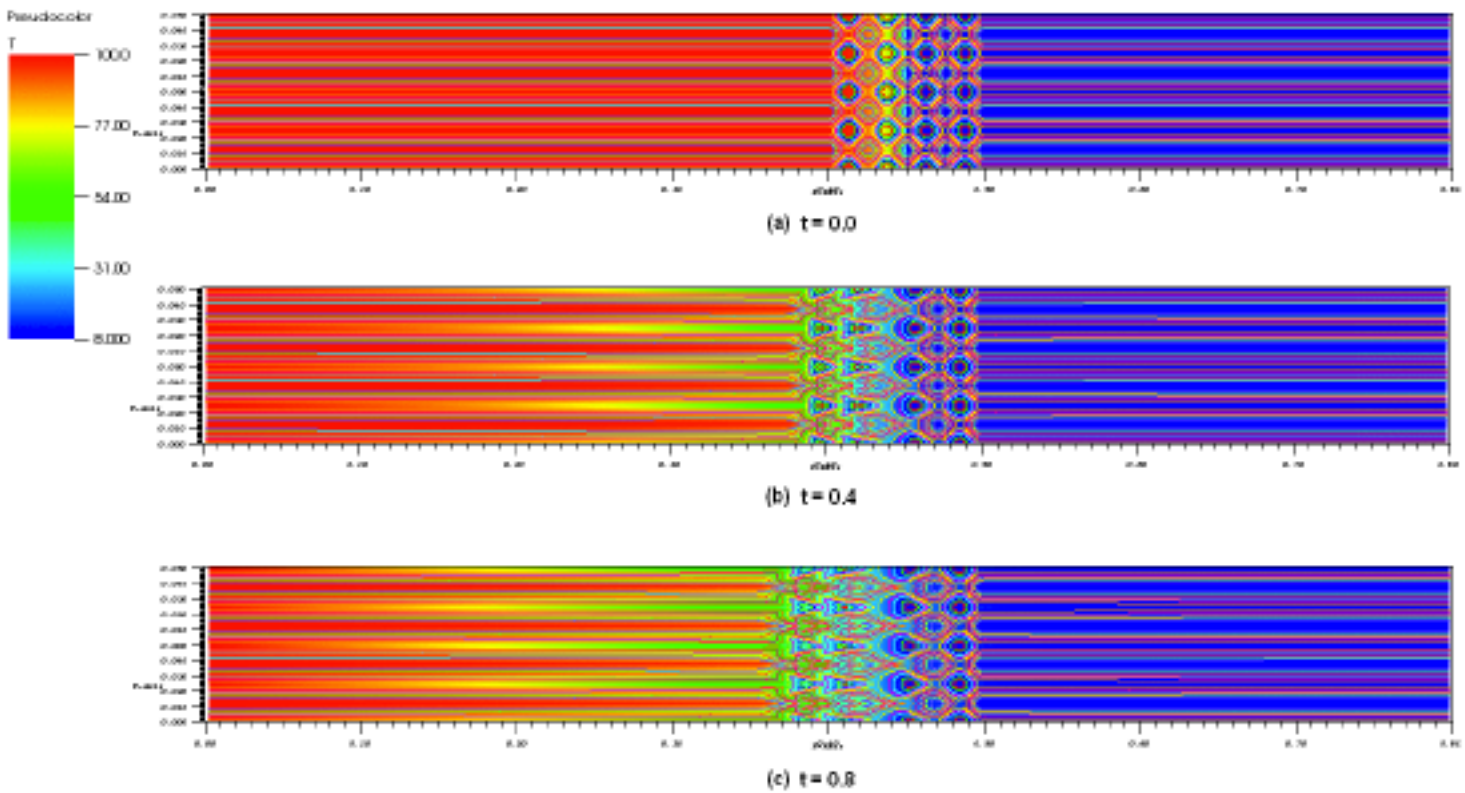
Because of the increased diffusivity at the origin, the field at the upper and lower half planes will be bent towards the origin, creating a reconnection spot.

The high reconnection rate at this reconnection spot converts the magnetic energy into kinetic energy and produces the observed flow pattern.

Anisotropic Heat Conduction (Li, Frank & Blackman 2012)

$$Q = -\kappa_{\parallel}(\nabla T)_{\parallel}$$

- Heat Conduction at Thermal Interfacce



Conclusions:

- AstroBEAR 2 AMR Multi-physics
  - Efficient AMR parallelization
  - MHD
  - Self-Gravity, Heat Conduction
  - Ionization Dynamics, Chemistry, Real EOS
  - Sink Particles
  - (X) Radiation Transfer
- Collaboration/Training
  - Wiki
  - Open for use by other groups (UNC, Rice, LANL, Bonn)