Intuitive approach to magnetic reconnection

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(Received 29 March 2011; accepted 3 August 2011; published online 6 September 2011)

Two reconnection problems are considered. The first problem concerns global physics. The plasma in the global reconnection region is in magnetostatic equilibrium. It is shown that this equilibrium can be uniquely characterized by a set of constraints. During reconnection and independently of the local reconnection physics, these constraints can be uniquely evolved from any initial state. The second problem concerns Petschek reconnection. Petschek's model for fast reconnection, which is governed by resistive MHD equations with constant resistivity is not validated by numerical simulations. Malyshkin *et al*. [Phys. Plasmas **12**, 102920 (2005)], showed that the reason for the discrepancy is that Petschek did not employ Ohm's law throughout the local diffusion region, but only at the X-point. A derivation of Petschek reconnection, including Ohm's law throughout the entire diffusion region, removes the discrepancy. This derivation is based largely on Petschek's original 1964 calculation [in *AAS-NASA Symposium on Solar Flares* (National Aeronautics and Space Administration, Washington, D.C., 1964), NASA SP50, p. 425]. A useful physical interpretation of the role which Ohm's law plays in the diffusion region is presented. © *2011 American Institute of Physics*. [doi:10.1063/1.3628312]

I. INTRODUCTION

There are two interesting problems concerning reconnection that I address in this paper. The first problem concerns the plasma dynamics in the region outside the reconnection layer, the global region. A proper treatment of the plasma in this region is necessary to derive proper boundary conditions for the reconnection layer.

For reconnection velocities that are not too fast compared with the Alfven speed, this region is in magnetostatic equilibrium. As magnetic reconnection of field lines transfers flux from one part of the global region to another, the global equilibria progress in a definite way that is independent of the physics in the reconnection layer. This was first shown by Uzdensky *et al*.,[12] making use of the variational principle of Kruskal and Kulsrud.[2] This progression puts a stronger constraint on the boundary conditions of the reconnection layer than is generally appreciated. Their proof of this variational principle is incomplete, and I present a complete proof in Sec. II.

The second interesting problem is: why is Petschek's fast reconnection not validated by numerical simulations? The reason has to do with the build-up of that component of the magnetic field which extends across the reconnection layer, and which supplies the necessary tension force for his model. This build-up is slower than Petschek supposed, leading to a longer diffusion region, and a slower reconnection rate than he predicts. Malyshkin *et al*.[5] mathematically explained this lack of validation by showing that the problem was in Petschek's incorrect application of Ohm's law in the diffusion region. In Sec. III, I show how this equation should have been treated, by repeating Petschek's original calculation. At the end of this section, I present a useful physical interpretation of the physics of the action of Ohm's law, in building up the transverse field in the diffusion region.

II. GLOBAL EQUILIBRIA

The geometry of the global region is indicated in Figure 1, which is a poloidal cross section of the reconnection geometry. In this paper, I assume that the reconnection geometry is toroidal, as it is in the Magnetic Reconnection Experiment (MRX). (One can also regard two-dimensional reconnection as toroidal if one identifies two different poloidal cross sections.) The local regions in the figure consist of the thin reconnection layer D and the separatrix layer E. The reconnection layer D is thin if the reconnection velocity is slow compared with the Alfven speed. Plasma flowing out of the thin reconnection layer flows into the separatrix layer at Alfvenic speeds, but its velocity is quickly dissipated by parallel viscosity in such a short time that only a small amount of flux is reconnected. Thus, the separatrix region is also thin. Therefore, global-reconnection theory can be considered to be a boundary-layer theory.

These local layers divide the rest of the geometry into three regions. In regions A and B the lines of force have not been reconnected. The lines in region C are the reconnected lines. Because the global region has a larger volume than the local region, the velocities in it are small, and the separate regions A, B, and C are each in separate magnetostatic equilibrium. Moreover, the total pressures, $p + B^2/8\pi$, on either side of D and E are equal, so that the three regions are in magnetostatic force balance with each other. I refer to the equilibrium of the entire region as the global equilibrium.

As the reconnection progresses, flux is transferred from A and B into C, and the global equilibrium changes. However, Uzdensky *et al*. showed, from a variational principle, that a set of constraints exist such that every global equilibrium is uniquely specified by the values taken by this set of

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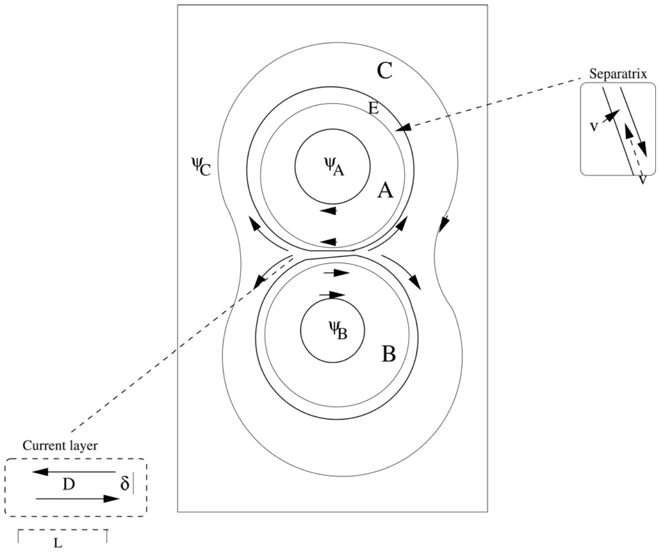

FIG. 1. Example of two cylinders.

constraints. Further, if the values of the set of constraints are known at an initial time, they can be found at any later time solely from the physics in the global region, and independently of the physics in the local region. This is really not so surprising, when one considers the problem as a boundary layer problem, since the global dynamic forces are larger than the transverse forces in the layers.

The example provided by shock conditions in hydrodynamics makes this plausible. The Rankine-Hugoniot conditions determine the hydrodynamics of the fluid outside of the shock region independently of the physics inside the shock layer.

A complete set of constraints which determine the equilibrium, exist under the assumption that the magnetic field in each of the three regions A, B, and C has magnetic surfaces. Let a magnetic surface in regions A, B, or C be labeled by ψ_A, ψ_B, or ψ_C, respectively. The $\psi's$ are equal to the poloidal fluxes included inside the surfaces. Under this assumption the constraints defined on each surface are:

- $M(\psi_A)$, $M(\psi_B)$, or $M(\psi_C)$, the mass included inside the surface.
- $\Phi(\psi_A)$, $\Phi(\psi_B)$, or $\Phi(\psi_C)$, the toroidal flux included inside the surface.
- $s(\psi_A)$, $s(\psi_B)$, or $s(\psi_C)$, the entropy $s = p/\rho^\gamma$ on the surface.

Now, choose three sets of functions, $M(\psi)$, $\Phi(\psi)$, $s(\psi)$, (defined differently in each of the three regions) to represent the constraints. Consider a state S_0, consisting of $\mathbf{B}(\mathbf{r})$, $p(\mathbf{r})$, and $\rho(\mathbf{r})$ defined throughout the global regions and such that these constraints are satisfied on each surface. Assume that the state S_0 has an energy

$$\mathcal{E} = \int \left(\frac{B^2}{8\pi} + \frac{p}{\gamma-1}\right) d^3\mathbf{x}, \qquad (1)$$

which is a minimum relative to the energy of any other state with the same values for the constraints. The integral is to be taken over all the global regions. This is to say, if one consider any other neighboring state S_1 with the same values for the constraints, its energy will be greater.

The set of constraints has been constructed so that, equality of their values for any two neighboring states, means it is possible to connect these two states by an ideal displacement $\boldsymbol{\xi}(\mathbf{r})$. That is to say, if one starts with the state S_0, and carries out this displacement, then it will be transformed into state S_1. The existence of this displacement will appear clear after a little thought. Its existence in each of the regions A, B, and C is rigorously established in the paper of Kruskal and Kulsrud.[2]

At a fixed point, $\boldsymbol{\xi}$ changes $\mathbf{B}$ by

$$\delta\mathbf{B} = \nabla \times (\boldsymbol{\xi} \times \mathbf{B}), \qquad (2)$$

and p by

$$\delta p = -\boldsymbol{\xi} \cdot \nabla p - \gamma p \nabla \cdot \boldsymbol{\xi}. \qquad (3)$$

Then the change in the energy $\mathcal{E}$, in the A region is

$$\delta\mathcal{E}_A = \int_A \left[\mathbf{B} \cdot \frac{\nabla \times (\boldsymbol{\xi} \times \mathbf{B})}{4\pi} - \frac{1}{\gamma-1}\boldsymbol{\xi} \cdot \nabla p - \frac{\gamma}{\gamma-1} p \nabla \cdot \boldsymbol{\xi}\right] d^3\mathbf{x} + \int_{S_A} \boldsymbol{\xi} \cdot \mathbf{n}\left(\frac{B^2}{8\pi} + \frac{p}{\gamma-1}\right) d^2\mathbf{x}. \qquad (4)$$

(The surface term takes into account, the fact that the two A regions of the two states may not be the same.) The sum of the changes in energies in the three regions A, B, and C must vanish for all choices of the displacement $\boldsymbol{\xi}(\mathbf{r})$.

One can transform the various terms in the above integral in the usual way by vector integration by parts. The result, keeping the integrated terms, is

$$\delta\mathcal{E}_A = \int_A \left[\boldsymbol{\xi} \cdot (-\mathbf{j} \times \mathbf{B} + \nabla p) - \nabla \cdot [\mathbf{B} \times (\boldsymbol{\xi} \times \mathbf{B})] - \frac{\gamma}{\gamma-1} \nabla \cdot (p\boldsymbol{\xi})\right] d^3\mathbf{x} + \int_{S_A} \boldsymbol{\xi} \cdot \mathbf{n}\left(\frac{B^2}{8\pi} + \frac{p}{\gamma-1}\right) d^2\mathbf{x}.$$

Now, apply Gauss's theorem to the divergence terms, and use the fact that $\mathbf{B} \cdot \mathbf{n} = 0$ on the surface S_A, to combine them with the other surface terms. The result is

$$\delta\mathcal{E}_A = \int_A \boldsymbol{\xi} \cdot (-\mathbf{j} \times \mathbf{B} + \nabla p) d^3\mathbf{x} - \int_{S_A} \boldsymbol{\xi} \cdot \mathbf{n}\left(p + \frac{B^2}{8\pi}\right) d^2\mathbf{x}. \qquad (5)$$

The energy of the state S_0 is to be a minimum with respect to that of the neighboring states with the same constraints. Therefore, $\delta\mathcal{E} = \delta\mathcal{E}_A + \delta\mathcal{E}_B + \delta\mathcal{E}_C$ must vanish for all $\boldsymbol{\xi}$. (Of course, any different choice of $\boldsymbol{\xi}$ corresponds to a different neighboring state S_1, but this is of no consequence since the different state still has a larger energy and satisfies the constraints.)

Take any point in A not on a boundary of A. Then one can choose $\boldsymbol{\xi}$ to be non zero and arbitrary, at and near that point, but vanishing elsewhere including on the boundary.

Then

$$\mathbf{j} \times \mathbf{B} = \nabla p, \tag{6}$$

at that point, and at every other point not on the boundary.

With this equation, the change in energy of region A reduces to

$$\delta \mathcal{E}_A = \int_{S_A} (\boldsymbol{\xi} \cdot \mathbf{n}) \left(p + \frac{B^2}{8\pi} \right) d^2\mathbf{x}. \tag{7}$$

Now, after adding the contributions to $\delta\mathcal{E}$ from regions B and C, the sums from the various surface terms will cancel, if and only if the jump in $p + B^2/8\pi$ vanishes over all the gaps between the regions. That is, across every point on D and E

$$\left\langle p + \frac{B^2}{8\pi} \right\rangle = 0. \tag{8}$$

This is because, the values of $\boldsymbol{\xi} \cdot \mathbf{n}$ are equal in magnitude and opposite in sign at every contact surface. Equations (6) and (8) show that the minimum energy state S_0 is a global equilibrium state.

Conversely, if the state S_0 is a magnetostatic state satisfying both Eqs. (6) and (8), then one can trace the argument backward to show that $\delta\mathcal{E} = 0$, and that S_0 has a minimum energy relative to any neighboring state satisfying the constraints.

At this point in the discussion, it is clear that for a given set of values of the constraints, $\Phi(\psi)$, $M(\psi)$, and $s(\psi)$, the minimum energy state is in global magnetostatic equilibrium, and the unique global magnetostatic equilibrium which has the specific values for the constraints. That is to say, given any values of the set of constraints, there is a unique equilibrium with these values. Therefore, there is a one-to-one correspondence between all values for our sets of constraints and all global magnetostatic equilibria. (Note that this set of constraints is a complete set, and this completeness is necessary for the one-to-one correspondence. If one constraint were missing, there would be an equilibrium state that is not a minimum energy state since, its energy could be lowered by a non ideal transformation to a neighboring equilibrium state that would satisfy all of the constraints except the missing one.)

How can one make use of this theorem, and how should one choose the values for the constraints? Notice that the constraints are ideal constants of the motion, and during any ideal evolution of the state, even a non-dynamic evolution, these constraints are conserved.

Consider the evolution of the global equilibria during a reconnection. The fluxes in regions A and B, decrease as the reconnection proceeds, but the values for the constraints as functions of the $\psi's$ in the two regions, remain the same over the range of $\psi's$, except at their limiting values near regions D and E. Similarly, the constraint functions in region C remain the same functions of its ψ over their range in region C, except at the limiting value near the separatrix. As the ranges in ψ_A and ψ_B decrease by an element of flux, the range of ψ_C increases by the same amount. Further, the value of Φ at the limit of the range of ψ_C is equal to the sum of its values at the limits of ψ_A and ψ_B.

However, the entropy at the maximum value of ψ_C, $s[\psi_C(max)]$, appears to be undetermined, because of the dissipative processes in the reconnection layers D and E. Does this mean that its value depends on the details of the physics in these layers? The answer is no. In the case that the *external boundary* of region C does not change, the total energy also cannot change. After the reconnection of an element of poloidal magnetic flux $\delta\psi$, the total energy must be the same. However, the regions D and E are so thin, that they contain a negligible amount of energy. Thus, it is only the total energy in the global region that must remain constant. But this energy is uniquely determined by the global magnetostatic equilibrium, which is determined by the values of the constraints. Remember that the only value of the constraints that is undetermined by the ideal evolution of the global equilibrium is the entropy $s[\psi_C(max)]$. Thus, energy conservation determines this one unknown value of the constraints. (Notice that there is an analogous situation in the hydrodynamic shock case; where the conservation of energy determines the jump in the entropy across the shock.)

It is remarkable that the sequence of global equilibria, which are controlled by the known sequence of the values of the constraints, is uniquely predetermined. The sequence is independent of the physics of the reconnection processes. This is true whether the reconnection model is that of Sweet-Parker,[6,10] Petschek,[7] or a model derived from two-fluid physics. This uniqueness arises because the values of the constraints can be evolved uniquely for the three regions, when energy conservation is used to determine the one value of the one constraint that does not evolve ideally.

It should be noted that, if the *external* boundary is moving, the total energy will change because work is done by the boundary. But this work is pdV work. Thus, the total global energy can be followed self-consistently independently of the rate at which the work is done. The amount of work done is independent of the rate of passage of the equilibria through the sequence, as long as the reconnection rate is slow compared to the rate of change of the external boundary. Thus, the unknown entropy constant $s[\psi_C(max)]$, can still be determined. The global energy at the time when a given equilibrium is reached depends only on the amount of external work done during this time. It is thus the same, no matter at what rate the work is done. Therefore, our conclusion as to the independence of the global equilibria of the local physics is still valid.

On the other hand, the time evolution of the ranges of ψ_A and ψ_B, (and correspondingly the evolution of the range of ψ_C) is entirely determined by the reconnection process, and *does* depend on the local physics.

For each of the magnetostatic equilibria in the sequence, the geometry and size of regions A, B, and C are determined. Thus, the positions and lengths of the regions D and E, are also determined. Further, $\mathbf{B}$ and p are known throughout the global regions, and in particular their values are known on the surfaces of D and E. Thus, at any given time, the local boundary conditions are determined by the corresponding global equilibrium in the sequence. This interplay between the global and local dynamics is exactly what is expected in a boundary-layer theory.

It is sometimes an advantage to view phenomena in reconnection physics from the global point of view. For example, if the reconnection is being forced by changing the external boundary, any pile-up of flux can be viewed as due to the changing global equilibrium, just as well as due to the lack of speed of the reconnection. In fact, unless the time for the change of the global equilibrium is slow enough to be comparable with the time to reconnect a finite amount of flux, the pile-up *should* be regarded as entirely a result of the changing global equilibria. In nature, the reconnection rate tends to be slow compared to the rate of evolution of the global equilibrium, but in laboratory experiments, such as the MRX, it may be comparable.

III. PETSCHEK RECONNECTION

As far as rapid magnetic reconnection in the large systems that occur in astrophysics and space physics is concerned, there is no question that a process such as that proposed by Petschek is of vital importance. No other mechanism except Petschek's comes close to producing fast reconnection in systems of such large size. Petschek's key idea is that resistive diffusion can be replaced by wave action in converting magnetic energy into other forms. The magnetic energy can be released by unfolding the reconnected lines (see Figure 2).

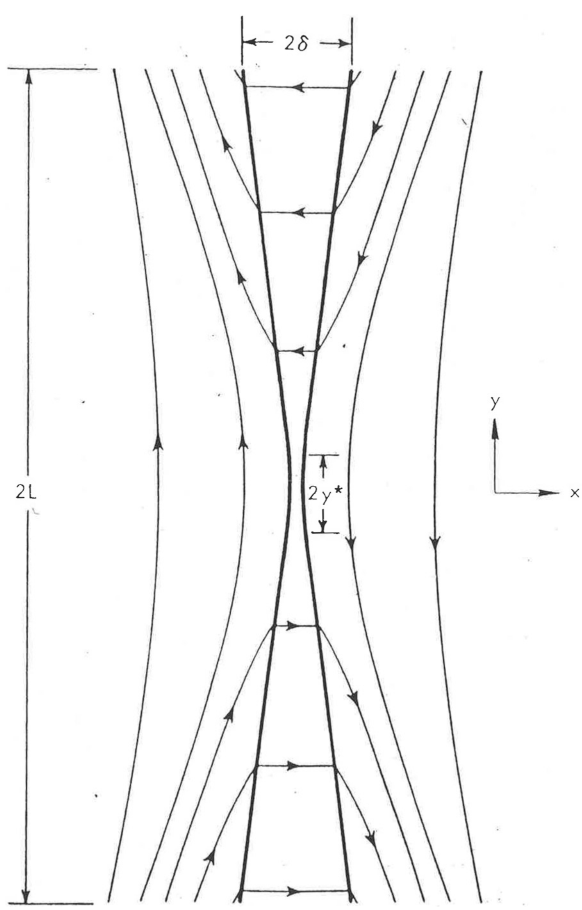

FIG. 2. Petschek's diagram.

In more detail, Petschek showed that if there is a magnetic field perpendicular to the reconnection layer, the incoming reconnection velocity can be balanced by a wave propagating along this field. Further, the tension force associated with this field accelerates plasma downstream faster than the flows driven by pressure gradients. This can allow a more rapid expulsion of the plasma from the reconnection layer than in the Sweet-Parker model.

Thus, it is disappointing that numerical simulations do not support the Petschek mechanism. The numerical simulations show that when resistive-MHD equations with constant resistivity apply, his mechanism does not lead to fast reconnection.[1,13] It is very important to intuitively understand why such a powerful reconnection mechanism fails, in order that a way can be found to restore it. An important advance was made towards this by Malyshkin *et al.*,[5] who showed that this failure is related to extra terms in Ohm's law which Petschek did not include.

The simplest way to explain the physics that is causing the problem is to trace through Petschek's original calculation.

In this section, I present his calculation in some detail, and follow it logically up to the point at which the additional terms in Ohm's law play a role. I then show that these terms restrict Petschek's conclusion: that his mechanism can lead to very fast reconnection. I then present a physical interpretation that shows why these terms are important.

To make the discussion clear, refer to Petschek's original diagram in Figure 2. Petschek takes his coordinate system with y in the outflow direction and x across the layer. The two important quantities in his theory are: the thickness of the reconnection layer, δ, and the transverse field, $B_x = b_x B_0$. These quantities are both functions of y.

The key point in his theory is that the incoming flux is balanced by wave propagation along the x axis as well as by resistive diffusion. Thus, his first equation is

$$M_0 = \frac{u_{x0}}{V_A} = \frac{\lambda}{V_A \delta} + |b_x|, \tag{9}$$

where $M_0 = u_{x0}/V_A$ is the Mach number of the reconnecting flow. I have introduced the notation $\lambda = c^2/4\pi\sigma$, with σ the conductivity.

For small y, say $y < y^*$, $|b_x|$ is small, and the incoming flow is mainly balanced by resistive diffusion. If y is large, ($y > y^*$) it is balanced by wave propagation. When $y > y^*$, $|b_x|$ is large enough to balance the incoming flow by itself, it becomes constant, and δ can increase. (In point of fact, Eq. (9) is only valid for very large and very small y. For y comparable to y^*, it is necessary to keep both terms in this equation. This is the nub of the difficulty with his model.)

Petschek next introduces the equation of continuity,

$$u_{x0} y = v(y)\delta(y), \tag{10}$$

where v is the flow in the y direction along the layer. The momentum equation is

$$\frac{d}{dy}(\rho v^2 \delta) = -\frac{B_0 B_x}{4\pi}. \tag{11}$$

Notice that B_x is negative in the upper part of his diagram, while u_{x0} is positive in the left side of his diagram. To keep the signs straight, I write the equations only for the upper left part of his diagram.

These equations are valid, both in the small-y diffusive region and the large-y wave propagation region. For simplicity, Petschek combines them into the single equation

$$M_0^2 \frac{d}{dy}\left(\frac{y^2}{\delta}\right) = -b_x. \tag{12}$$

Now, at large y, the diffusive term in Eq. (9) can be neglected, and

$$M_0 = -b_x, \quad y > y^*. \tag{13}$$

Substitution of this equation into Eq. (12) yields the important result

$$\delta(y) = M_0 y, \quad y > y^*. \tag{14}$$

This equation shows that the width of the outflow channel increases with distance y.

Next, in the diffusive region, $y < y^*$, the first term of Eq. (9) shows that

$$\delta = \frac{\lambda}{V_A M_0}, \qquad y < y^*, \tag{15}$$

is a constant.

Equations (10) and (15) show that the velocity v increases linearly with y to keep up with the incoming flow. Its acceleration is produced by magnetic tension, so that $-b_x$ must also increase linearly with y. From Eq. (12)

$$b_x = -\frac{2M_0^3 V_A}{\lambda} y, \quad y < y^*. \tag{16}$$

Now, when y reaches y^*, $|b_x|$ will have grown to the point at which wave propagation takes over the balancing of the incoming flow. That is, at $y = y^*$, $|b_x|$ must equal M_0. Thus,

$$y^* = \frac{\lambda}{2V_A M_0^2}. \tag{17}$$

For any given Mach number of the incoming flow M_0, the length of the diffusion region, y^* is given by this equation. Now, since all of Petschek's equations are satisfied, it was reasonable for him to suppose that any reconnection velocity can be accommodated by the proper choice of y^*. (There actually is a logarithmic limit on the reconnection rate which Petschek emphasized, but I here ignore.)

Before proceeding further, note that the above equations reduce to those of Sweet-Parker in the diffusive region. In fact, the solutions for δ, v, and b_x in the diffusion region, can be written in terms of y^* as

$$\delta = \frac{\lambda}{u_{x0}}, \quad v = \frac{1}{2}\frac{y}{y^*} V_A, \tag{18}$$

and

$$b_x = -\frac{u_{x0}}{V_A}\frac{y}{y^*}. \tag{19}$$

These equations are exactly those of the Sweet-Parker model with the global length set to y^*.

Now consider the z component of the full Ohm's law in the diffusive region,

$$cE - v b_x B_0 = \frac{c j_z}{\sigma} = -\frac{\lambda}{\delta}\left(B_0 + B_0'' \frac{y^2}{2}\right), \tag{20}$$

where the primes on B_0 denote y derivatives. The first terms on the left and right hand sides cancel. Making use of Eqs. (18) and (19) for v and b_x, the remaining terms in this equation are:

$$-v b_x B_0 = \frac{u_{x0} B_0}{2}\left(\frac{y}{y^*}\right)^2 = -\frac{\lambda B_0''}{2\delta} y^2 = -\frac{u_{x0} B_0''}{2} y^2, \tag{21}$$

or

$$\frac{B_0}{y^{*2}} = -B_0''. \tag{22}$$

But B_0 is the reconnecting field just outside of the layer, so that B_0'' is determined by the global equilibrium. (This, as has been shown in Sec. II is independent of the reconnection process.) Thus, its scale must be the global length L, and

$$B_0'' = \frac{d^2 B_0}{dy^2} \approx -\frac{B_0}{L^2}. \tag{23}$$

Therefore, from Eq. (22), $y^* \approx L$. This means that Petschek's assumption, that $y^* \ll L$ is inconsistent with the *complete* set of equations.

From Eq. (17), with $y^* \approx L$,

$$u_{x0}^2 \approx \frac{\lambda v_A}{2L}, \tag{24}$$

the Sweet-Parker result. This explains why numerical simulations of Petschek reconnection find the Sweet-Parker reconnection rate.

A physical interpretation of the role of Ohm's law is: that the term vB_x represents the rate at which the B_x field lines are swept downstream, while the cj''/σ term is the rate at which the incoming B_y field lines rotate into B_x field lines. This picture of the motion of the field lines was suggested in two earlier papers.[3,4]

This interpretation can be seen more quantitatively by rewriting Ohm's law as

$$\mathbf{E} + \frac{(\mathbf{v} + \mathbf{v}_{slip}) \times \mathbf{B}}{c} = 0, \tag{25}$$

in which I introduce the slip velocity $\mathbf{v}_{slip}$ to replace the resistive term, by defining it to satisfy

$$\eta \mathbf{j} = -\frac{\mathbf{v}_{slip} \times \mathbf{B}}{c}, \tag{26}$$

so that $\mathbf{v}_{slip} = \eta c \mathbf{j} \times \mathbf{B}/B^2$.

In this form of Ohm's law it is clear that, in the diffuse region a field line moves with the plasma velocity, **v**, plus the slip velocity, $\mathbf{v}_{slip}$. Hence, a field line is transported by the plasma velocity v in the y direction, and sheared in the x direction by the y gradient of $v_{slip,x}$. The scale of the slip velocity is of order the global scale L, and its magnitude is of order the reconnection velocity u_{x0}.

Thus, the line will be sheared, or rotated, at the rate

$$\frac{d\theta}{dt} = \frac{dv_{slip,x}}{dy} \approx \frac{u_{x0} y}{L^2}, \tag{27}$$

where $\theta = b_x$ is the angle the line of force makes with the y axis. At the same time, it will be transported in the y direction at the velocity

$$\frac{dy}{dt} = v = \frac{y}{2y^*} v_A. \tag{28}$$

Upon dividing these two equations

$$\frac{d\theta}{dy} = \frac{2u_{x0} y^*}{L^2 V_A}, \tag{29}$$

so that the line of force rotates through an angle $\Delta\theta \approx b_x (y^*/L)^2$. $\Delta\theta = b_x$ is the change in θ that Petschek needs for his theory. Thus, under this interpretation, the diffusion region must have a global length, to be long enough to allow sufficient rotation to generate the required downstream field B_x.

So far, I have restricted the discussion to a constant spatial resistivity. If the resistivity is non constant and varies on a shorter scale, L_η, than L, then the shear in $v_{slip,x}$ is increased by L/L_η, and the Petschek theory applies with the shorter length $y^* \approx L_\eta$. This result is in accord with the numerical simulations in Refs. 8, 9, and 11. It is of considerable interest to apply these intuitive ideas, which involve field line motion, to other reconnection models.

To summarize, Petschek's error was to apply Ohm's law only for $y = 0$ and $y > y^*$, ignoring it in the region $0 < y < y^*$. It is in this region that the build-up of the B_x field occurs. Petschek simply assumed this build-up could be arbitrarily fast because he found no equation restricting it. By properly including the full Ohm's law in this region, the rate of build-up of B_x, and thus y^*, are determined. As a consequence, the reconnection rate is slower than the "Petschek" rate.

IV. CONCLUSION

In the first part of this paper, I have shown that: reconnection physics in the global regions can be investigated more systematically by making use of a variational principle, to uniquely specify the global equilibrium in terms of a set of values for a set of constraints. The evolution of these values then gives the evolution of its equilibrium. I show that this evolution proceeds independently of the local physics in the reconnection and separatrix layers. In the second part, I have shown why the Petschek mechanism acts differently in numerical simulations than Petschek might have expected, and I give a physical interpretation to intuitively explain why this happens. This interpretation should be explored more fully to determine whether it can lead to other new physics in reconnection theory.

ACKNOWLEDGMENTS

I am grateful to Leonid Malyshkin for the suggestion that the problem with the Petschek model of reconnection, lies in the neglect of extra terms in Ohm's law. I also would like to thank Dmitri Uzdensky for reading the paper and making a number of useful suggestions. In addition, I owe the referee a vote of thanks for forcing me to do a creditable job in presenting this material in a more coherent form than resulted from my first attempt. Support for research on these problems was provided by the Center for Magnetic Self Organization (CMSO).

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