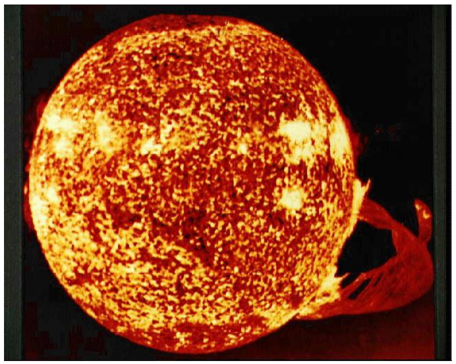
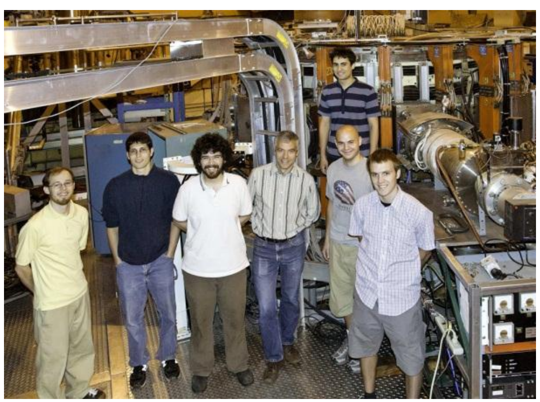
Role of Pressure Anisotropy in Reconnection





J Egedal, A Le,
O Ohia, A Vrublevskis,
W Daughton, H Karimabadi
& VS Lukin

MIT, PSFC, Cambridge, MA

PIC simulations

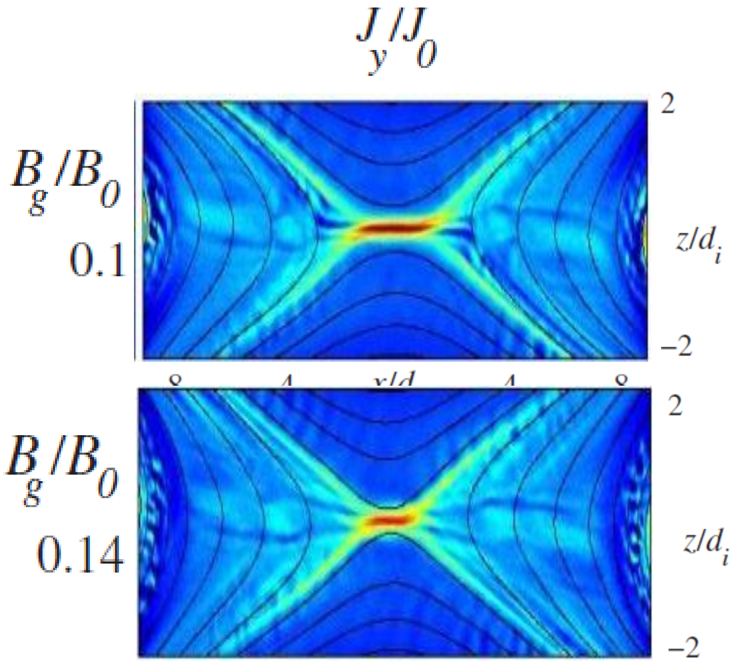
Fluid simulations

Massachusetts Institute of Technology

THE UNIVERSITY of WISCONSIN MADISON

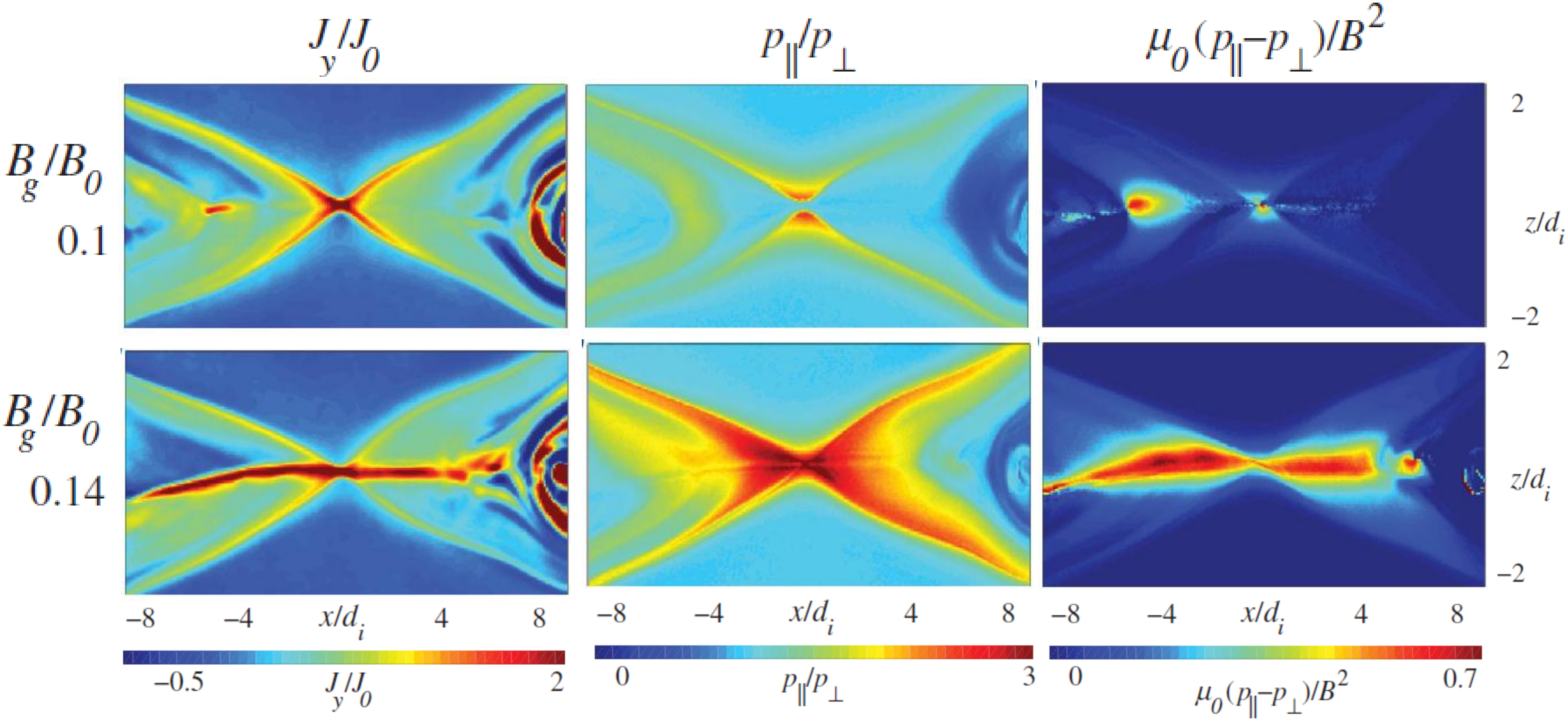
Pressure Anisotropy Important to the Structure of the Reconnection Region

Kinetic simulation results at $m_i / m_e = 400$, [A Le et al., PRL 2013]



Pressure Anisotropy Important to the Structure of the Reconnection Region

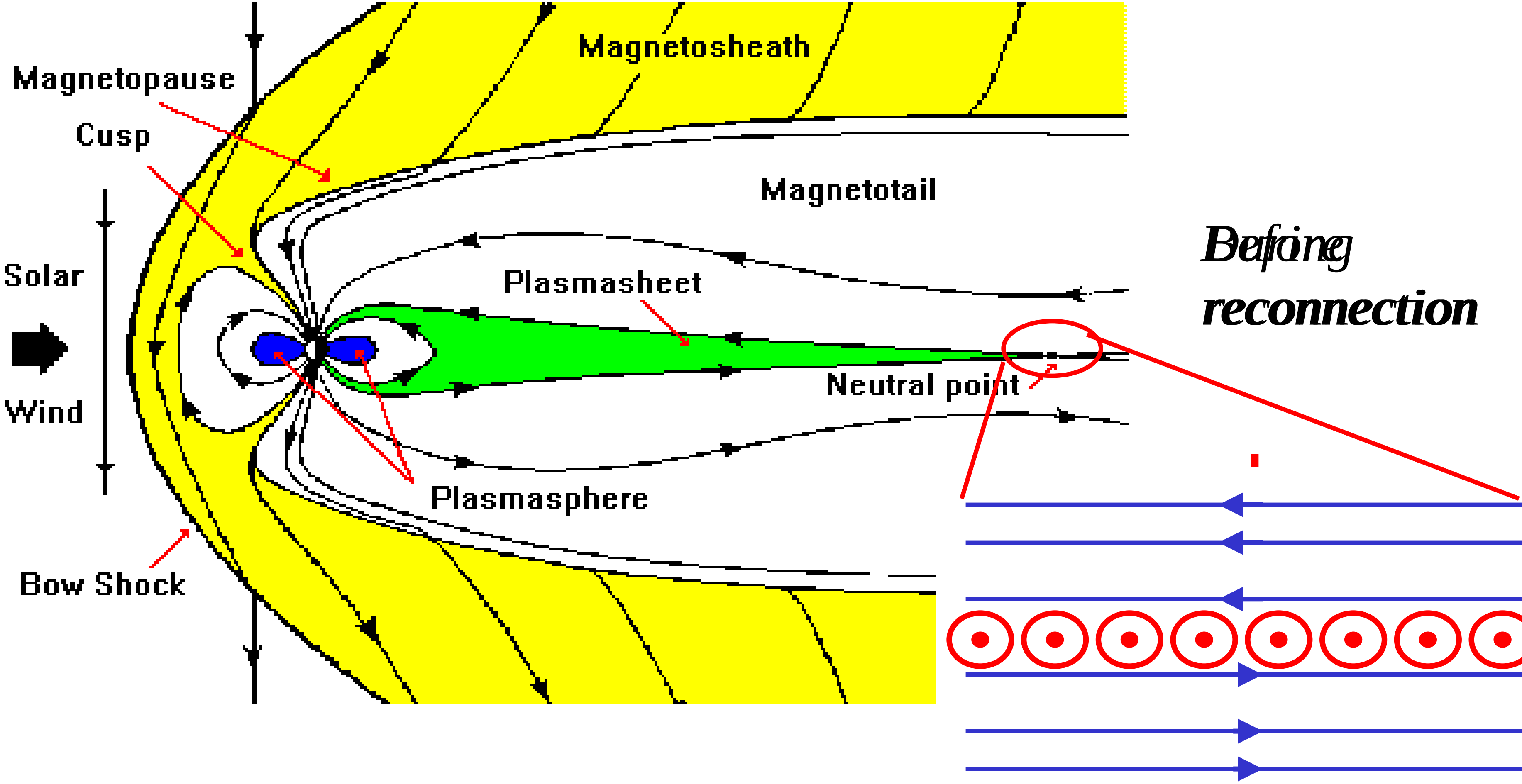
Kinetic simulation results at $m_i / m_e = 1836$, [A Le et al., PRL 2013]



Outline

- Spacecraft observations of electron distributions
- Kinetic model for electrons anisotropy using $\Phi_{\|}$
- Magnetized electron Equations of State (EoS)
- Force balance of the electron diffusion region
- Regimes of the electron diffusion region
- Conclusions

The Earth's Magnetic Shield



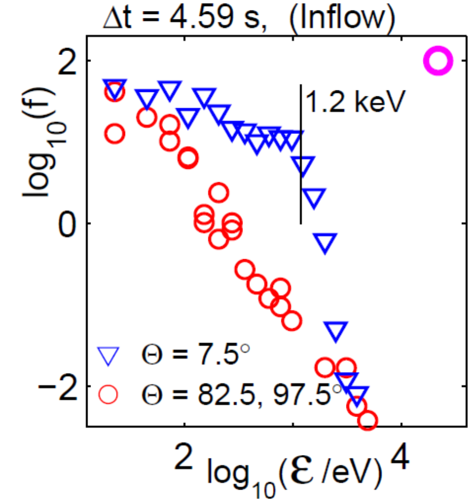
Model for Inflow Anisotropy

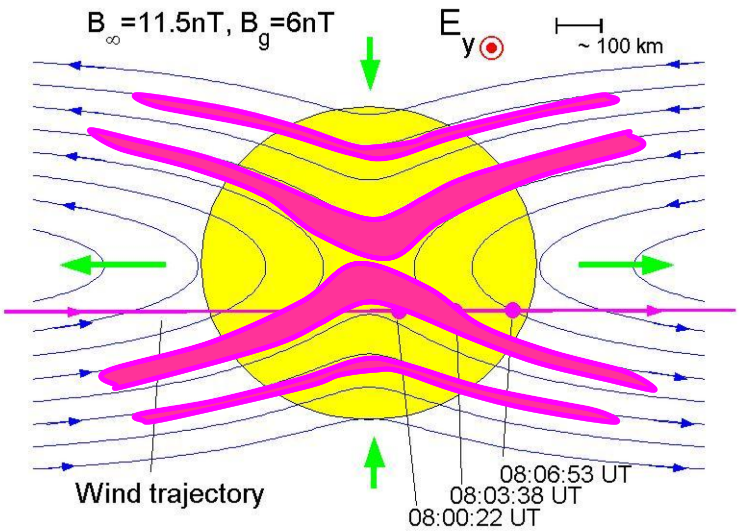
- Measurements within the ion diffusion region reveal:
  Strong anisotropy in f_e

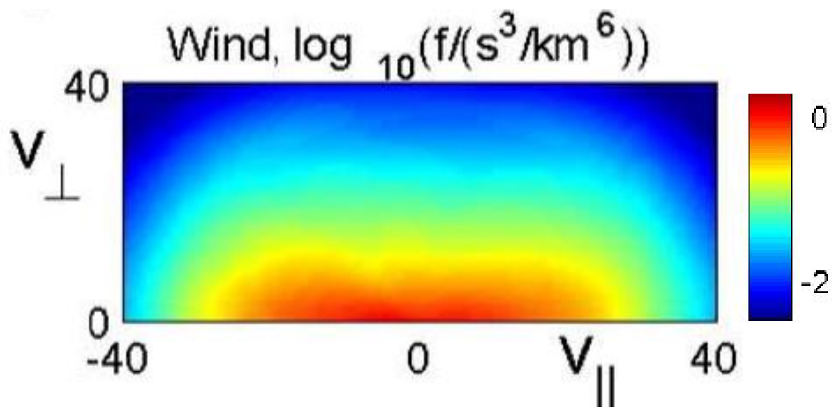
  $p_{||} > p_{\perp}$

Cluster observations 2001-10-01.

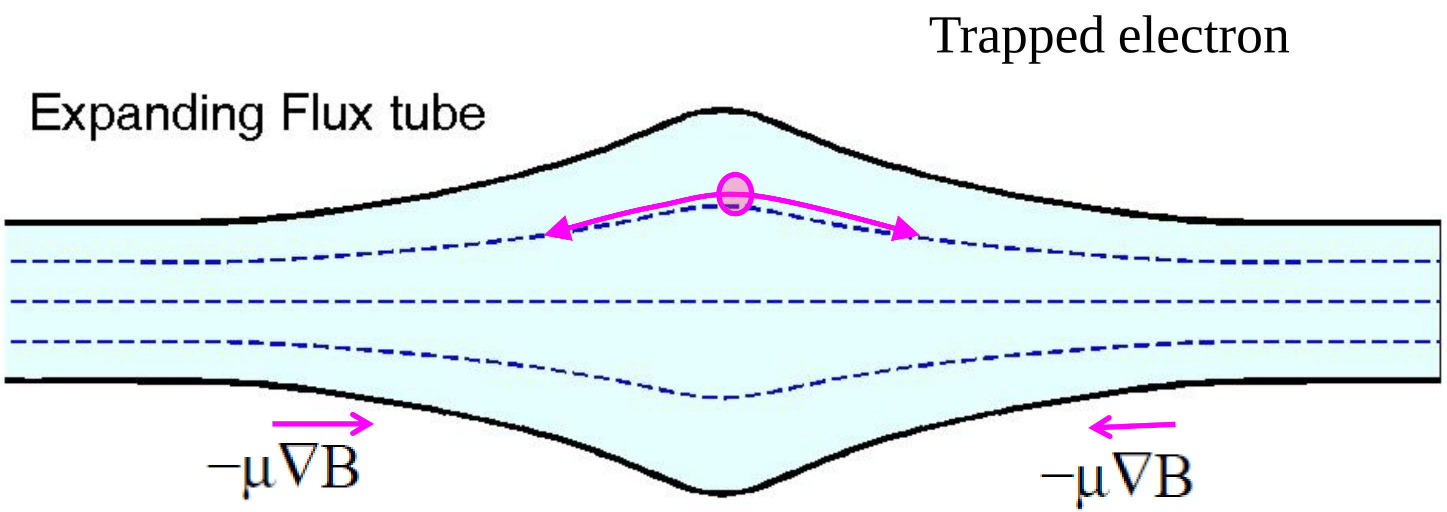
Bi-directional Beams







Electrons in an Expanding Flux Tube

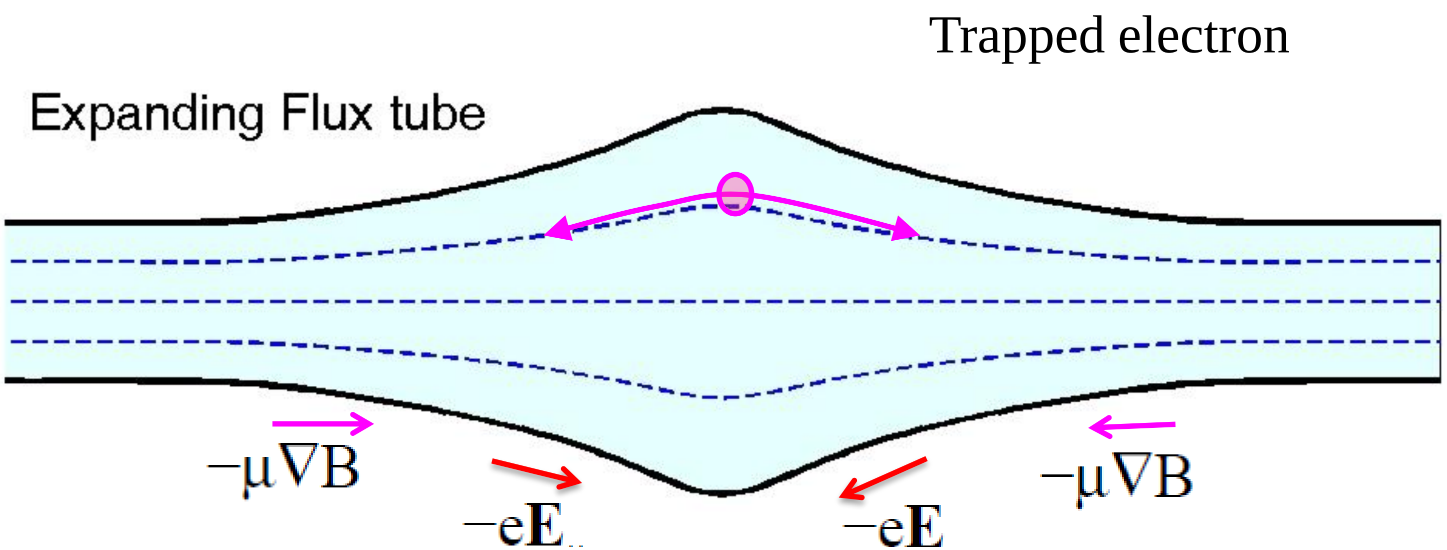


Magnetic moment:

$$\mu = \frac{m v_\perp^2}{2B}$$

➔ mirror force:

Electrons in an Expanding Flux Tube

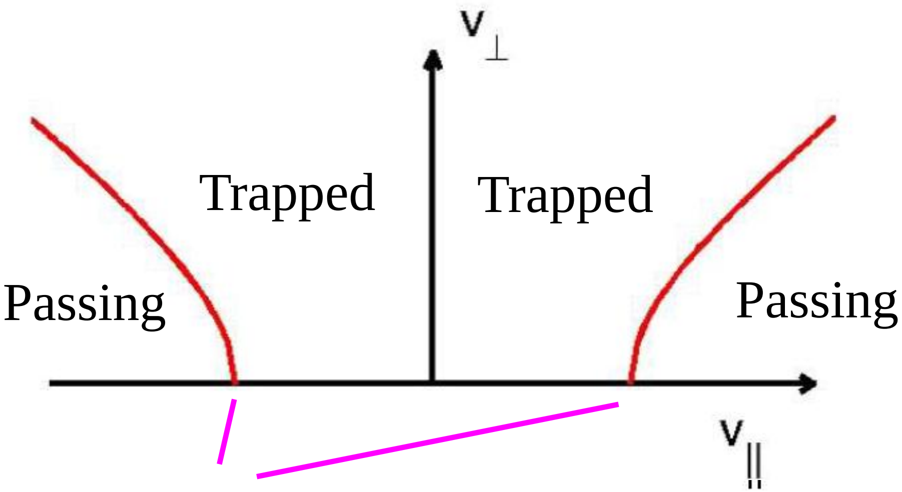


Magnetic moment:

$$\mu = \frac{m v_\perp^2}{2B}$$

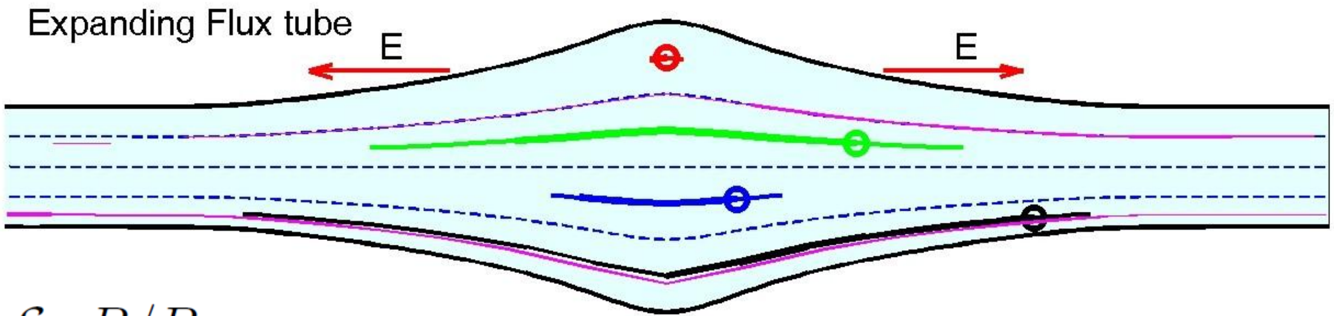
➔ mirror force:

$$\Phi_\parallel(\mathbf{x}) = \int_{\mathbf{x}}^{\infty} \mathbf{E} \cdot d\mathbf{l}$$



$$0.5 m v_\parallel^2 = e\Phi_\parallel$$

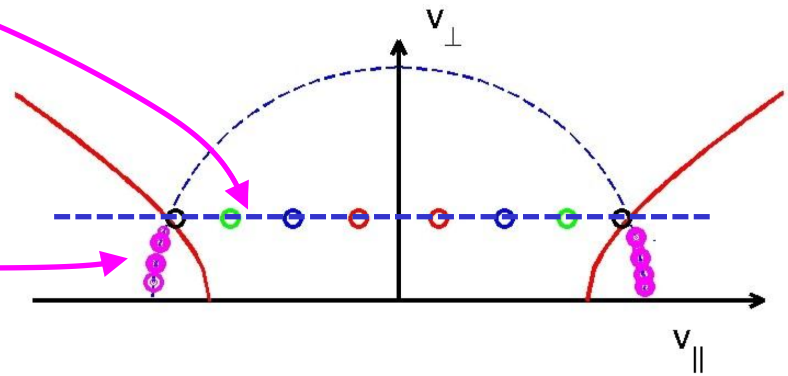
Electrons in an Expanding Flux Tube



Trapped:

$$\mathcal{E}_\perp = \mu B = \mathcal{E}_\infty B / B_\infty$$

$$\Longrightarrow \quad \underline{\mathcal{E}_\infty = \mu B_\infty}$$



Passing:

$$\mathcal{E} = \mathcal{E}_\infty + e\Phi_{||}$$

$$\Longrightarrow \quad \underline{\mathcal{E}_\infty = \mathcal{E} - e\Phi_{||}}$$

Vlasov:

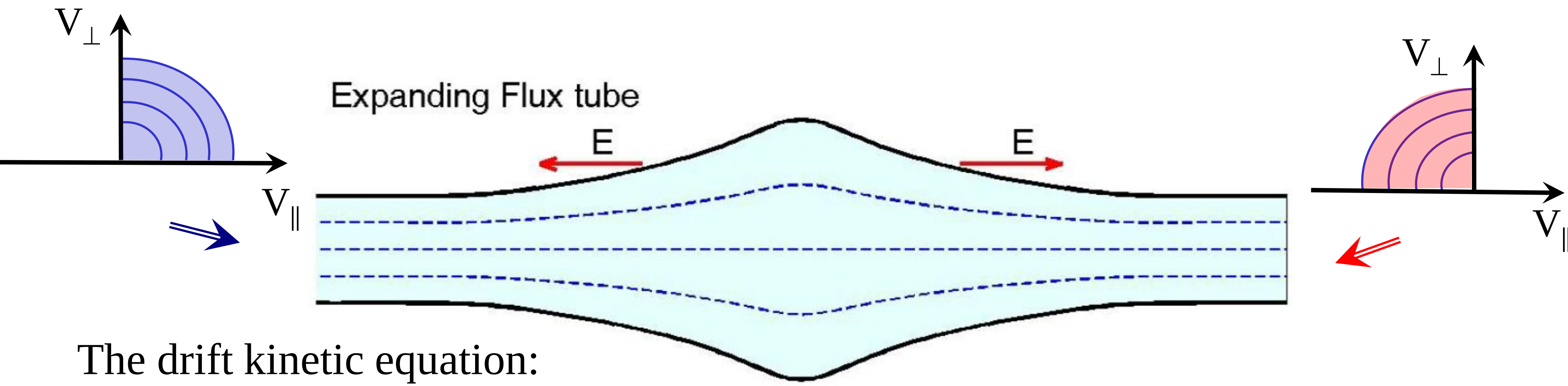
$$\frac{df}{dt} = 0$$

$$f(\mathbf{x}, \mathbf{v}) = f_\infty(\mathcal{E}_\infty)$$

$$f(\mathbf{x}, \mathbf{v}) = \begin{cases} f_\infty(\mathcal{E} - e\Phi_{||}) & , \quad \text{passing} \\ f_\infty(\mu B_\infty) & , \quad \text{trapped} \end{cases}$$

J. Egedal et al., JGR (2009)

Formal Derivation using an "Ordering"



The drift kinetic equation:

$$\frac{\partial f}{\partial t}+(\mathbf{v}_\parallel+\mathbf{v}_D)\cdot\nabla f+\left[\mu\frac{\partial B}{\partial t}+e(\mathbf{v}_\parallel+\mathbf{v}_D)\cdot\mathbf{E}\right]\frac{\partial f}{\partial \mathcal{E}}=0$$

Boundary conditions:

$$B = B_\infty$$

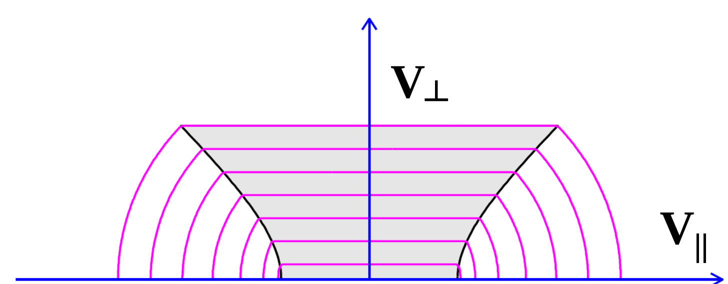
$$f = f_\infty(\mathcal{E}_{\parallel\infty}, \mathcal{E}_{\perp\infty})$$

Ordering:

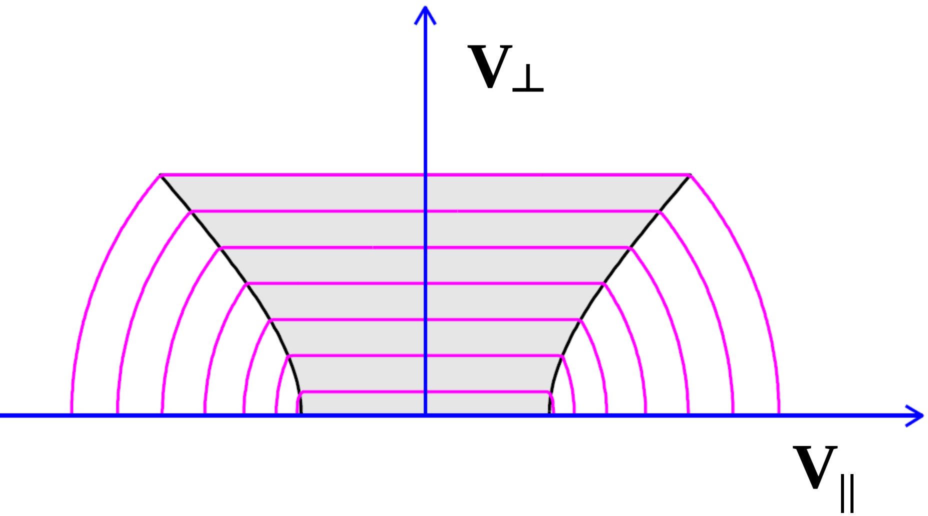
$$\nabla_\parallel \sim \frac{1}{L} \quad , \quad \nabla_\perp \sim \frac{1}{d} \quad , \quad \frac{\partial}{\partial t} \sim \frac{v_D}{d}$$

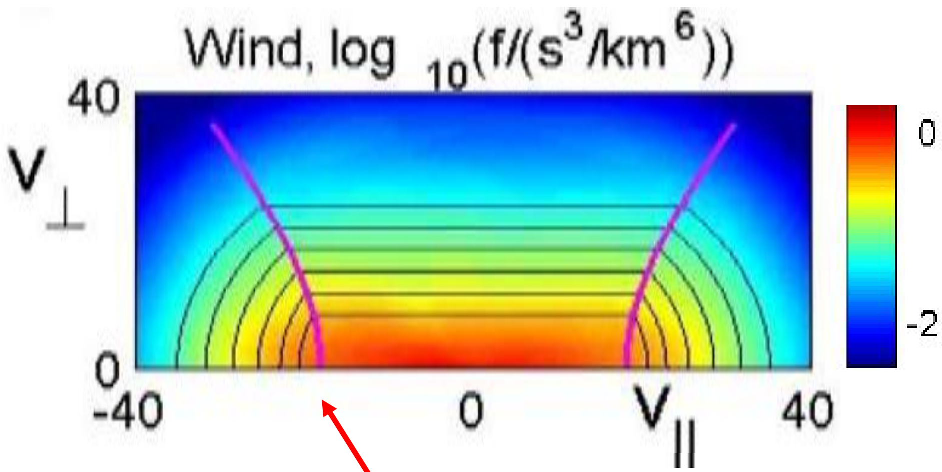
$$\frac{d}{L} \sim \delta \quad , \quad \frac{v_D}{v_t} \sim \delta^2 \qquad f = f_0 + \delta f_1 + \ldots$$

$$f_0(\mathbf{x},\mathbf{v}) = \begin{cases} f_\infty(\mathcal{E} - e\Phi_\parallel) & , \text{ passing} \\ f_\infty(\mu B_\infty) & , \text{ trapped} \end{cases}$$



Wind Spacecraft Observations in Distant Magnetotail, $60R_E$



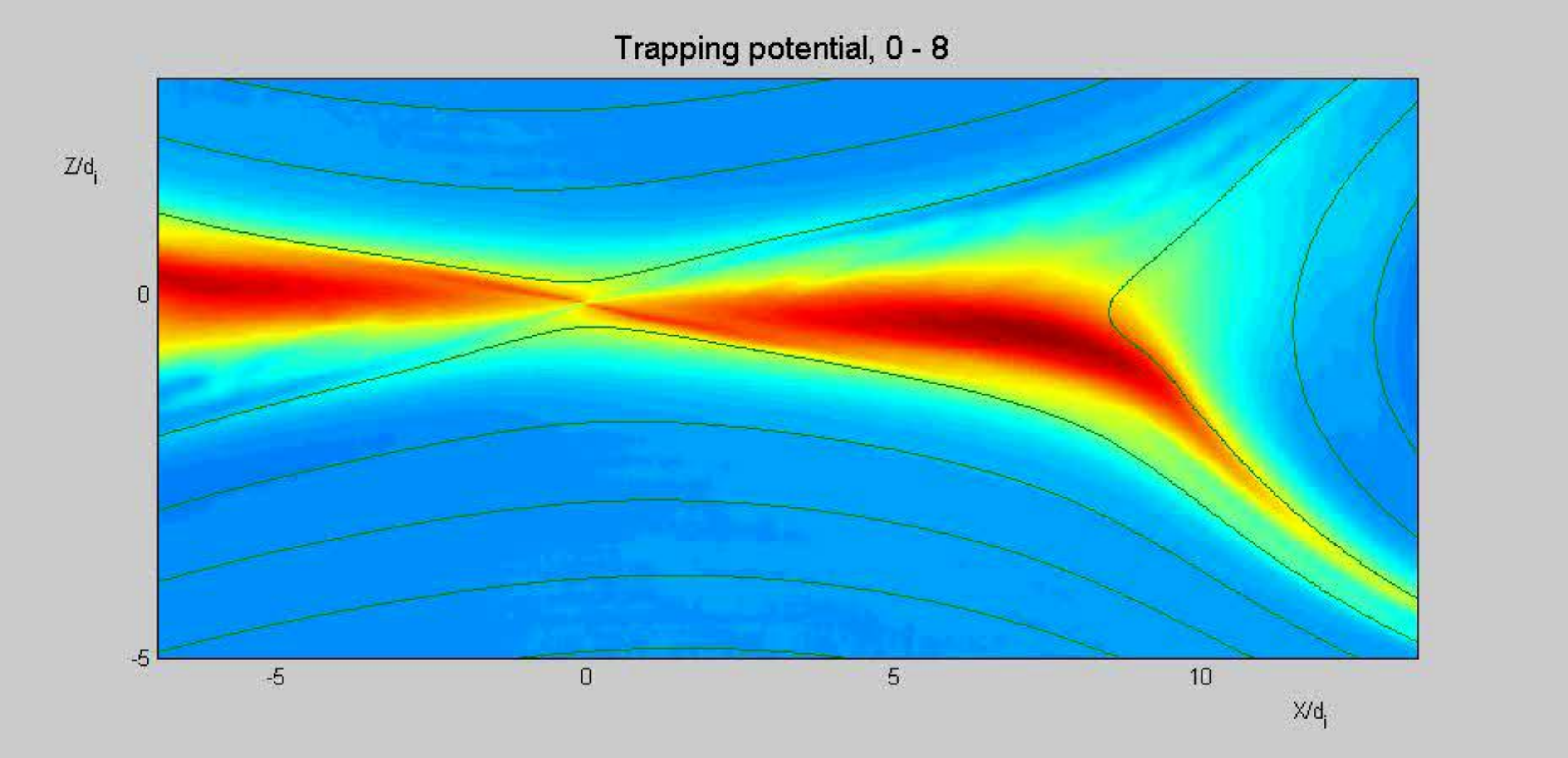


$$f(\mathbf{x},\mathbf{v}) = \begin{cases} f_\infty(\mathcal{E} - e\Phi_{||}) & , \text{ passing} \\ f_\infty(\mu B_\infty) & , \text{ trapped} \end{cases}$$

$\Phi_{||} \sim 1$ kV

The Acceleration Potential in a Kinetic Simulation

$e\Phi_{||} / T_e$



Kinetic Model ➔ Fluid Closure (EoS)

$$f(\mathbf{x},\mathbf{v}) = \begin{cases} f_\infty(\mathcal{E} - e\Phi_{||}) & , \text{ passing} \\ f_\infty(\mu B_\infty) & , \text{ trapped} \end{cases}$$

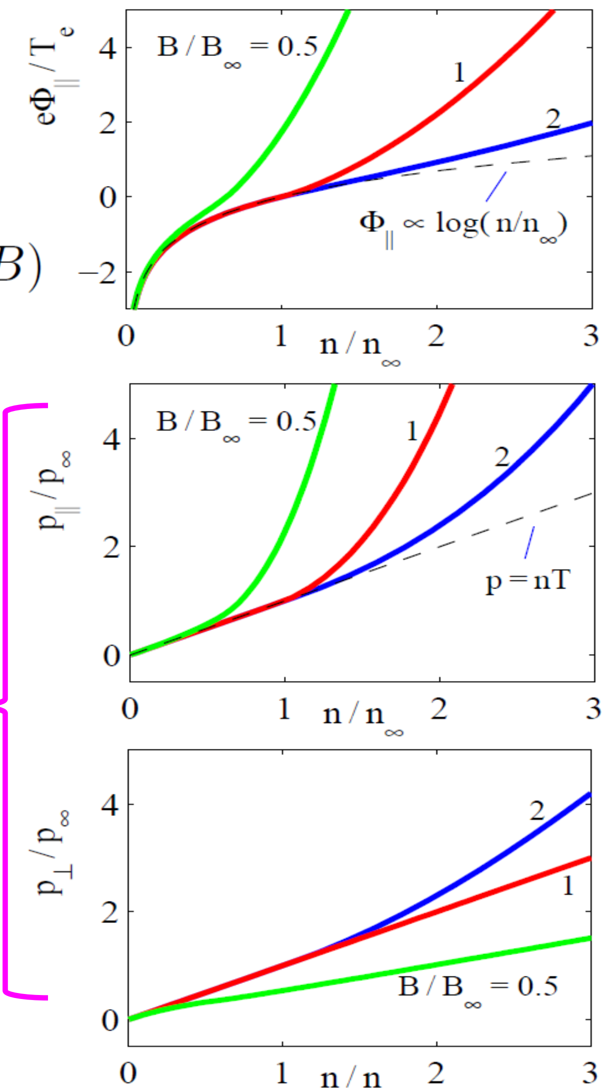
$\int \ldots d^3v$

$$\begin{cases} n = n(B,\Phi_{||}) \\ p_{||} = p_{||}(B,\Phi_{||}) \\ p_\perp = p_\perp(B,\Phi_{||}) \end{cases}$$

$$\Phi_{||} = \Phi_{||}(n,B)$$

Eliminate $\Phi_{||}$ ➔

$$\begin{cases} p_{||} = p_{||}(n,B) \\ p_\perp = p_\perp(n,B) \end{cases}$$



➔ $\Phi_{||} \gg T_e$

$$p_{||} \propto \frac{n^3}{B^2}$$

$$p_\perp \propto nB$$

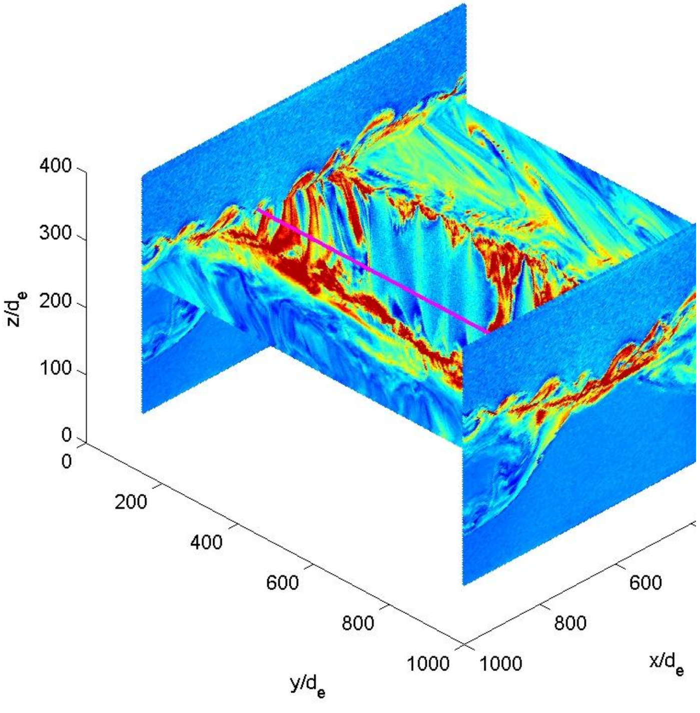
Transition from Boltzmann to double adiabatic CGL-scaling
[G Chew, M Goldberger, F E Low, 1956]

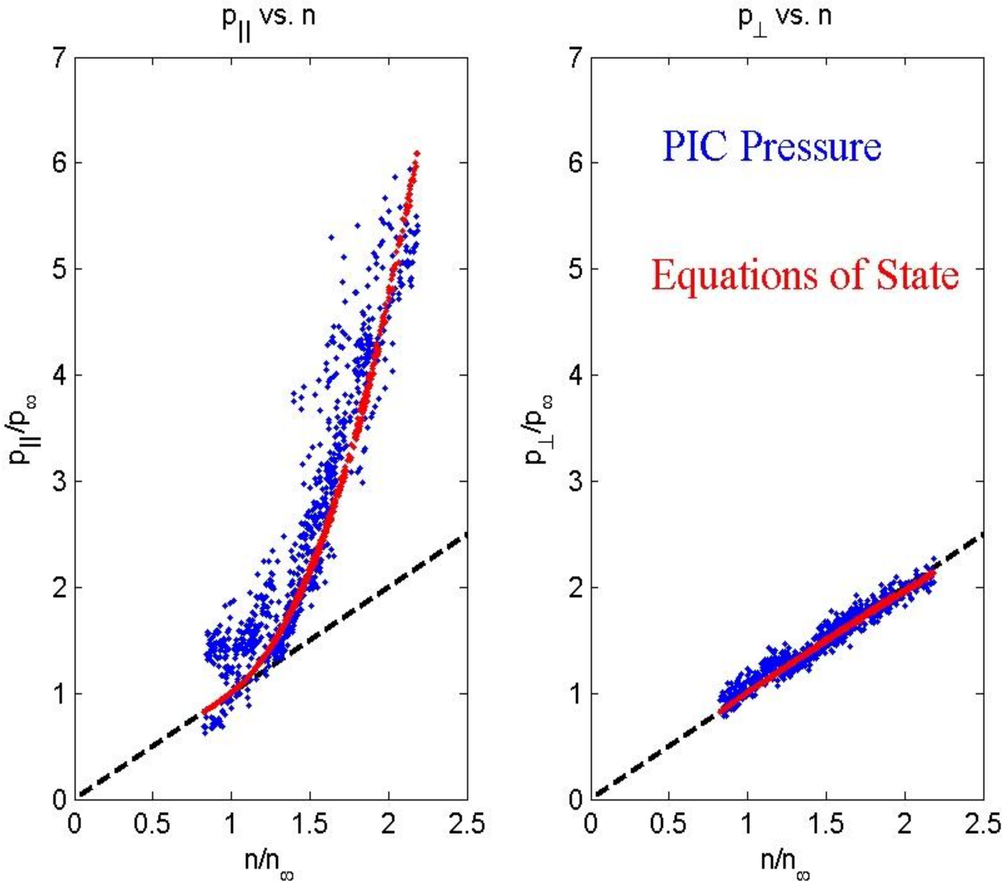
A. Le et al., PRL (2009)

Confirmed in Kinetic Simulations

EoS previously confirmed in 2D simulations,
now also in 3D simulations.

$p_{\|} / p_{\perp}$





New EoS Now Implemented in Two-Fluid Code

New code implemented by O Ohia using the HiFi framework developed in part by VS Lukin

Standard two-fluid equations

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{V}_i) = 0$$

$$m_i n \left(\frac{\partial \mathbf{V}_i}{\partial t} + \mathbf{V}_i \cdot \nabla \mathbf{V}_i \right) = \mathbf{J} \times \mathbf{B} - \nabla \cdot \bar{\mathbf{P}} + m_i n \nu_i \nabla^2 \mathbf{V}_i$$

$$\frac{\partial}{\partial t}\left(\frac{p_i}{n^\Gamma}\right) = -\mathbf{V}_i \cdot \nabla \frac{p_i}{n^\Gamma}$$

$$\frac{\partial \mathbf{B}'}{\partial t} = -\nabla \times \mathbf{E}'$$

$$\mathbf{E}' + \mathbf{V}_i \times \mathbf{B} = \frac{1}{ne}\left(\mathbf{J} \times \mathbf{B}' - \nabla \cdot \bar{\mathbf{P}}_e\right) + \eta_R \mathbf{J} - \eta_H \nabla^2 \mathbf{J}$$

$$\mathbf{B}' = \left(1 - d_e^2 \nabla^2\right) \mathbf{B}$$

$$\mu_0 \mathbf{J} = \nabla \times \mathbf{B}$$

Anisotropic pressure model

$$\bar{\mathbf{P}} = p_i \bar{\mathbf{I}} + \bar{\mathbf{P}}_e = p_i \bar{\mathbf{I}} + p_\perp \bar{\mathbf{I}} + \left(p_\parallel - p_\perp\right) \frac{\mathbf{B}\mathbf{B}}{B^2}$$

$$\tilde{p}_\parallel = \tilde{n} \frac{2}{2+\alpha} + \frac{\pi \tilde{n}^3}{6\tilde{B}^2} \frac{2\alpha}{2\alpha+1}$$

$$\tilde{p}_\perp = \tilde{n} \frac{1}{1+\alpha} + \tilde{n}\tilde{B} \frac{\alpha}{\alpha+1}$$

where $\alpha = \tilde{n}^3/\tilde{B}^2$ and for any quantity Q, $\tilde{Q} = Q/Q_\infty$

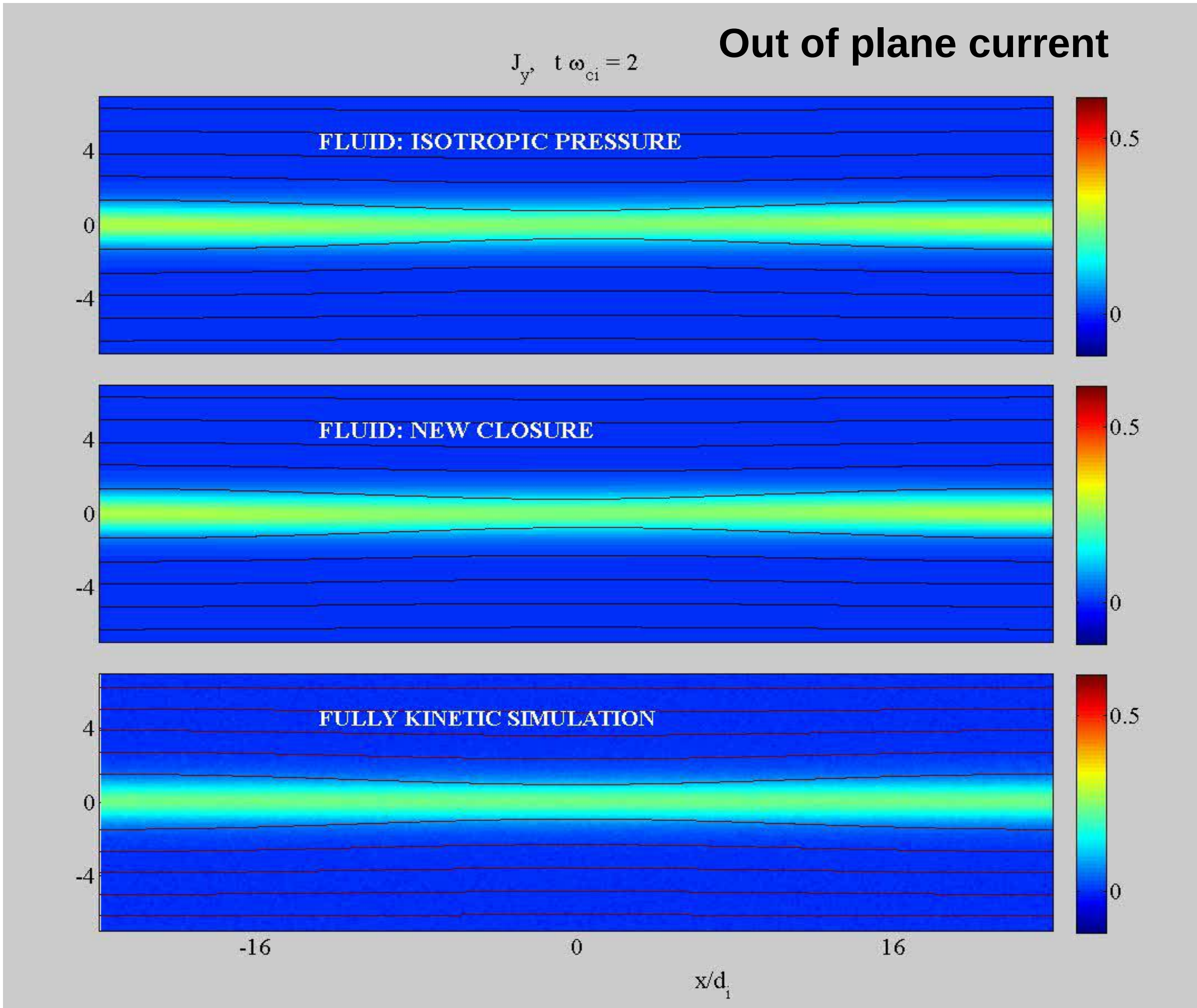
New *EoS* Implemented in Two-Fluid Code

Ohia, et al. PRL 2012 (using the HiFi framework by V.S. Lukin)

Isotropic pressure

Anisotropic pressure

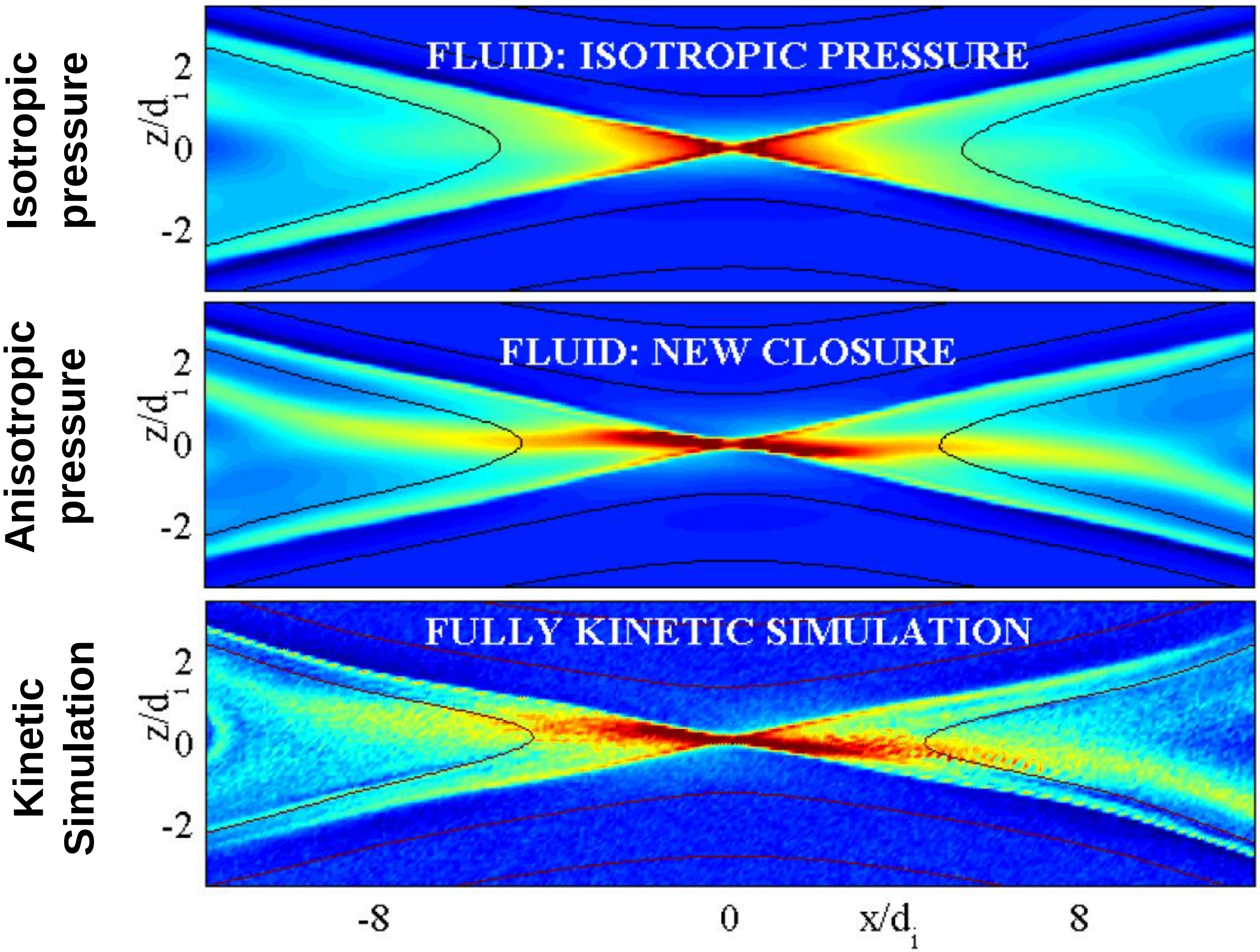
Kinetic Simulation



New *EoS* Implemented in Two-Fluid Code

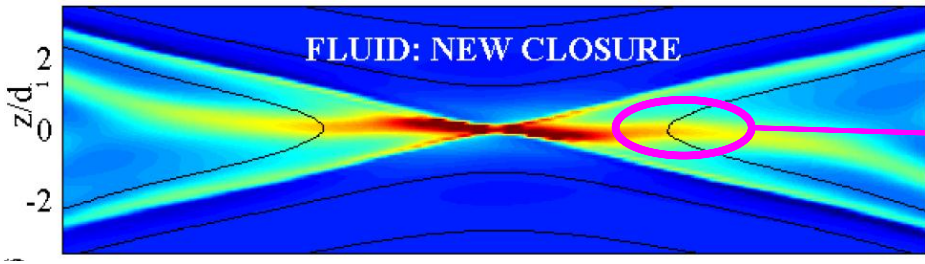
Ohia, et al. PRL 2012 (using the HiFi framework by V.S. Lukin)

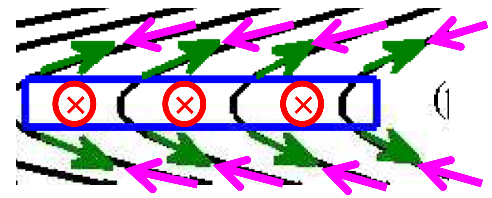
Out of plane current



Analytic Model for Electron Jets

Anisotropic pressure





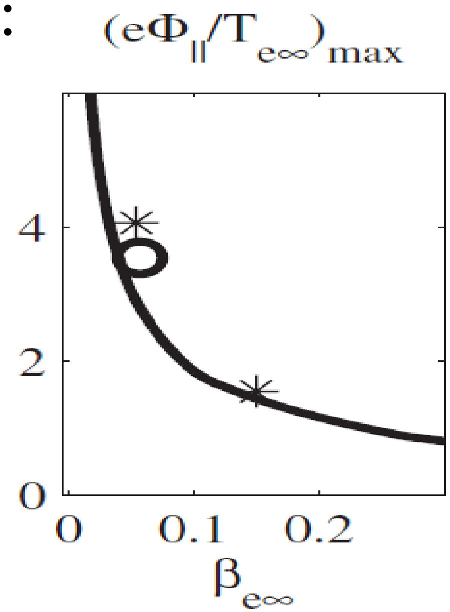
$p_{\|} - p_{\perp}$

Additional current term:

$$J_{\perp \text{extra}} = [(p_{\|} - p_{\perp})/B]\hat{b} \times \hat{b} \cdot \nabla \hat{b}$$

The magnetic tension is balanced by pressure anisotropy:

$$p_{\|}(n, B) - p_{\perp}(n, B) = B^2/\mu_0$$

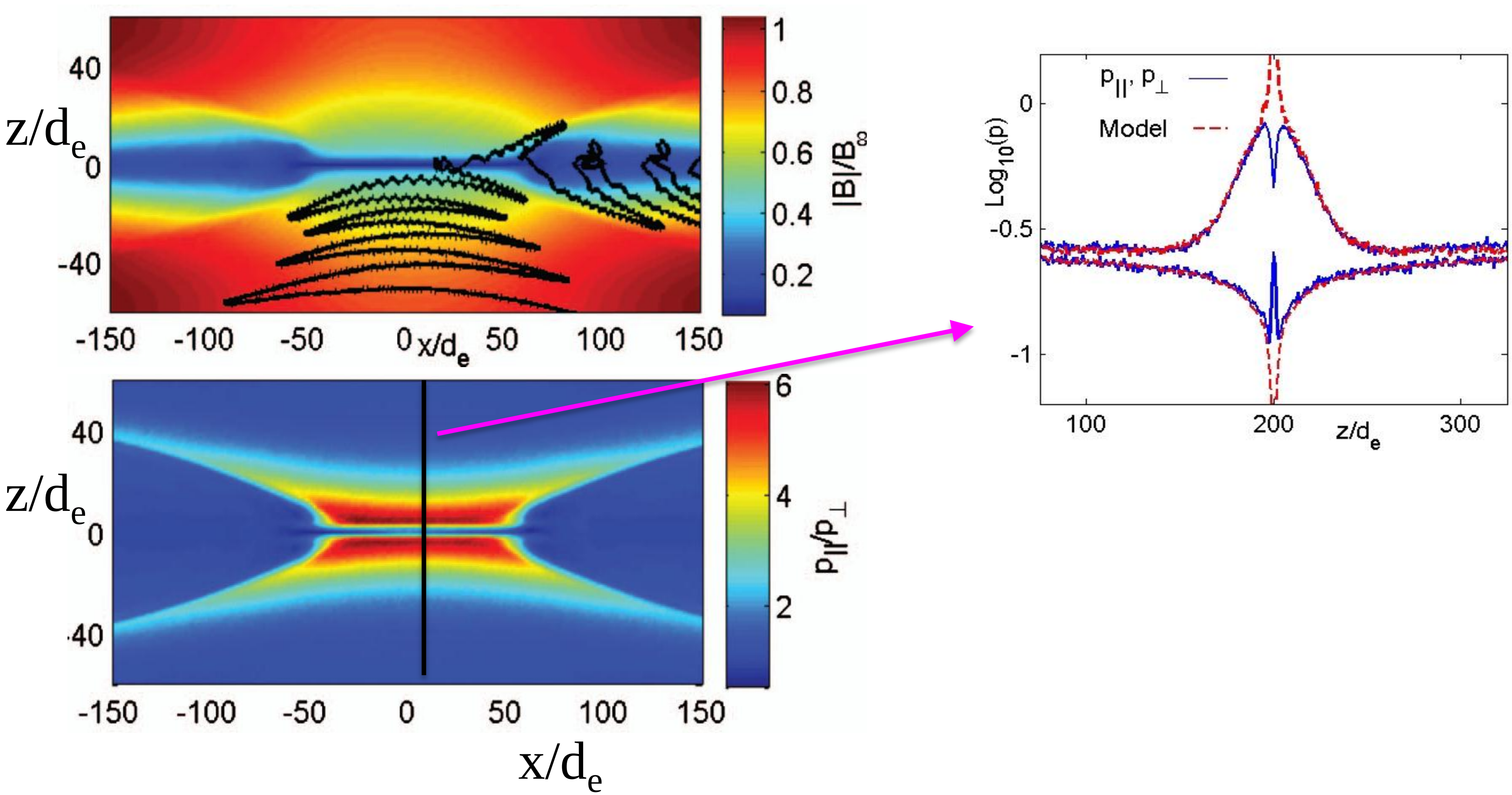


Use *EoS* to get scaling laws:

$$\beta_{e\infty} = \frac{\text{plasma pressure}}{\text{magnetic pressure}}$$

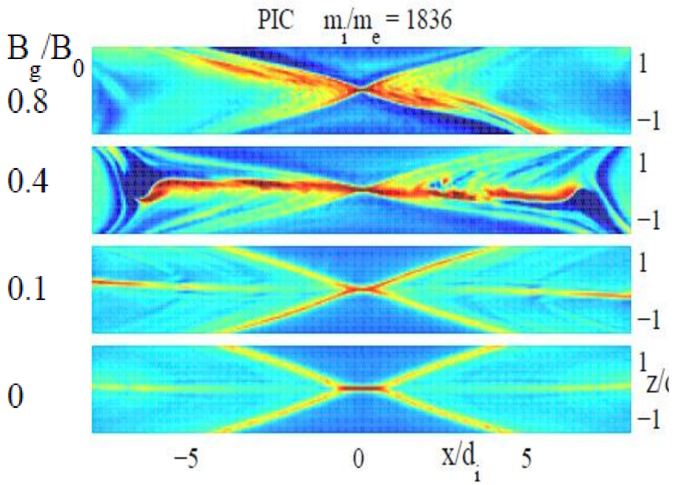
EoS for Anti-Parallel Reconnection?

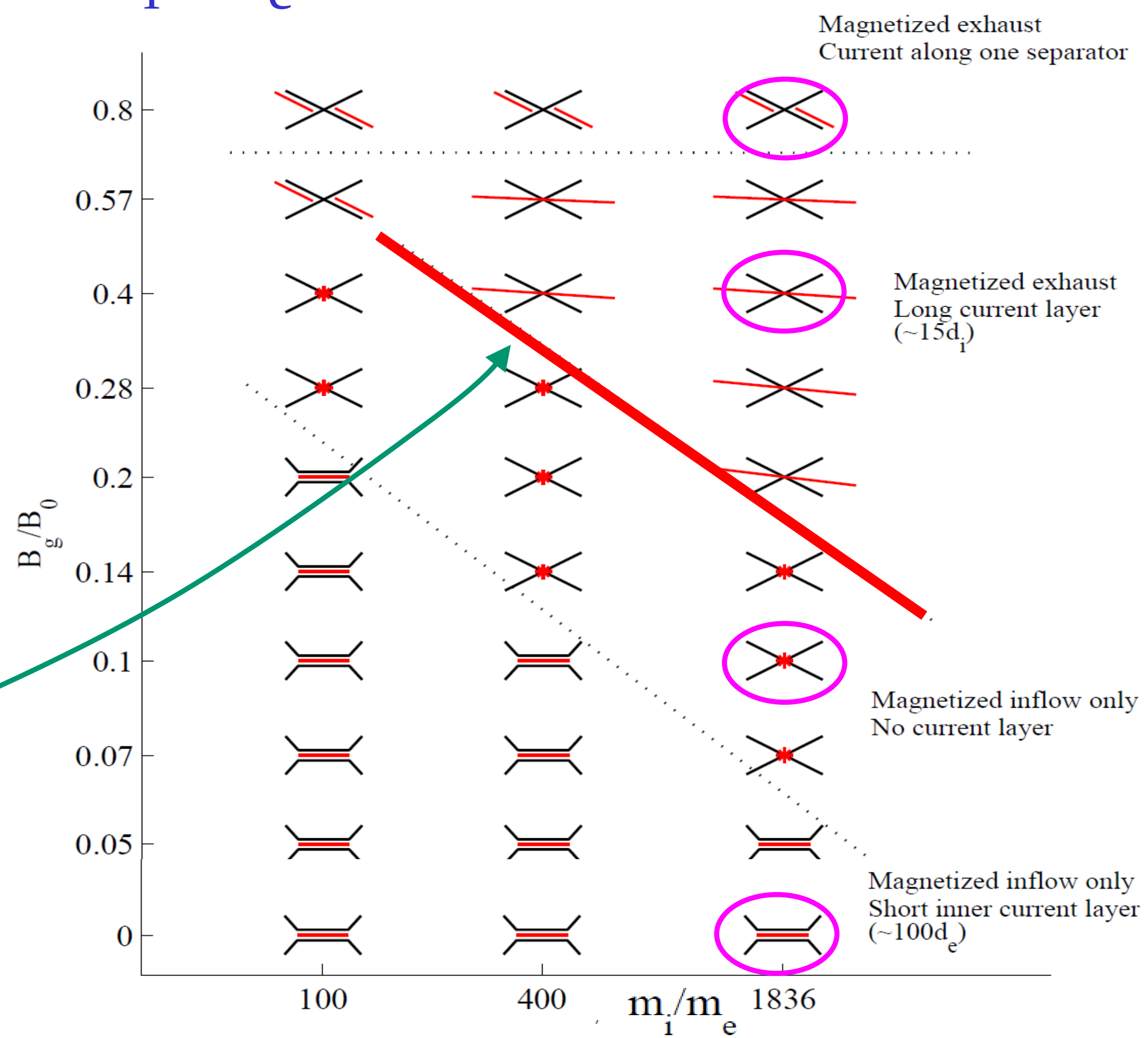
The electrons are magnetized in the inflow region:



Scan in B_g and m_i/m_e

($\beta_e = 0.03$)





Pitch angle diffusion

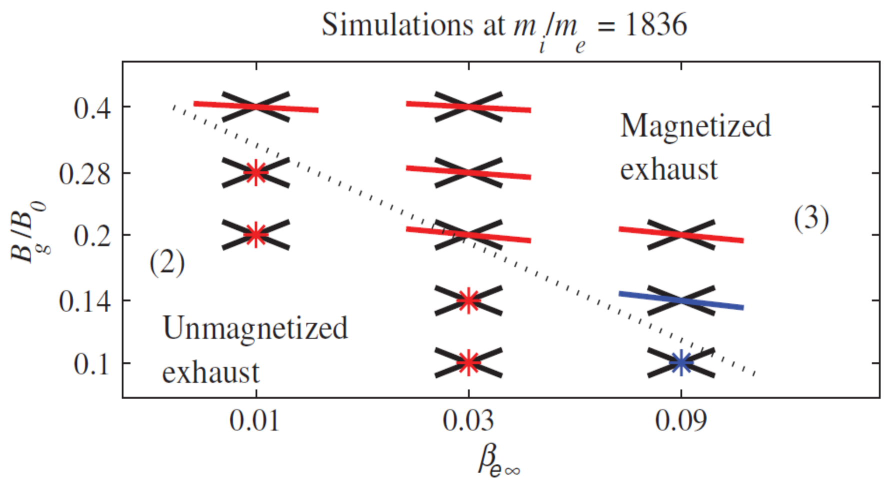
Pitch angle diffusion is controlled by:

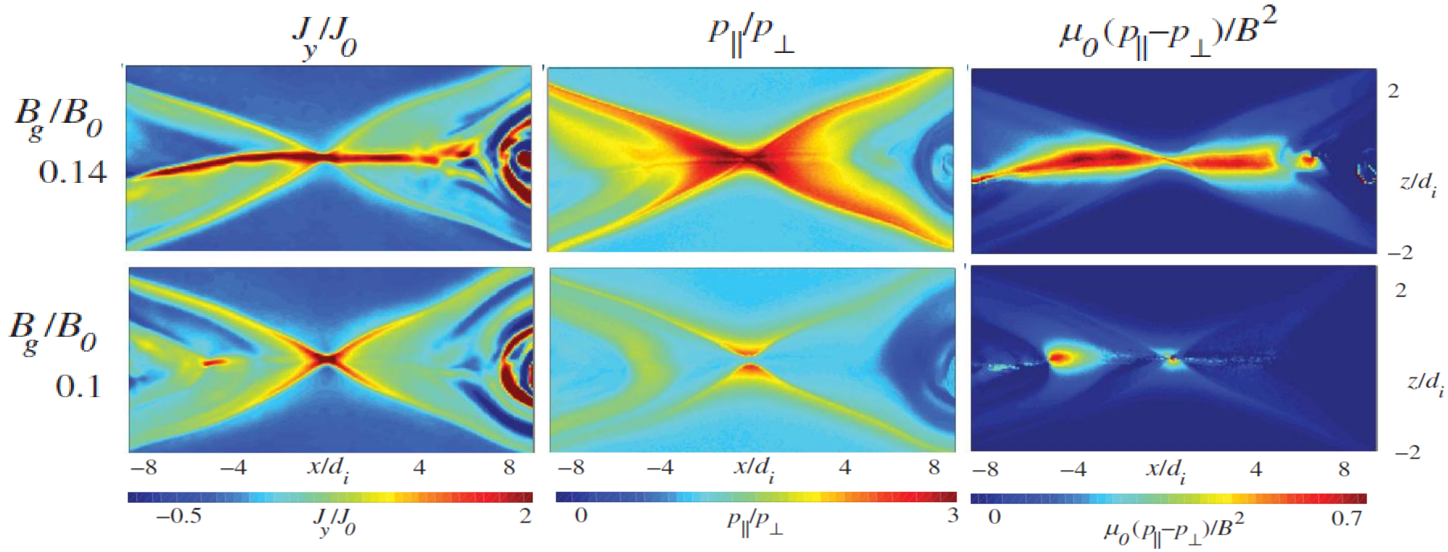
$$\kappa = \sqrt{R_B/\rho_e}$$

Depends on B_g and m_i/m_e

Summary:
Pressure Anisotropy is Important!

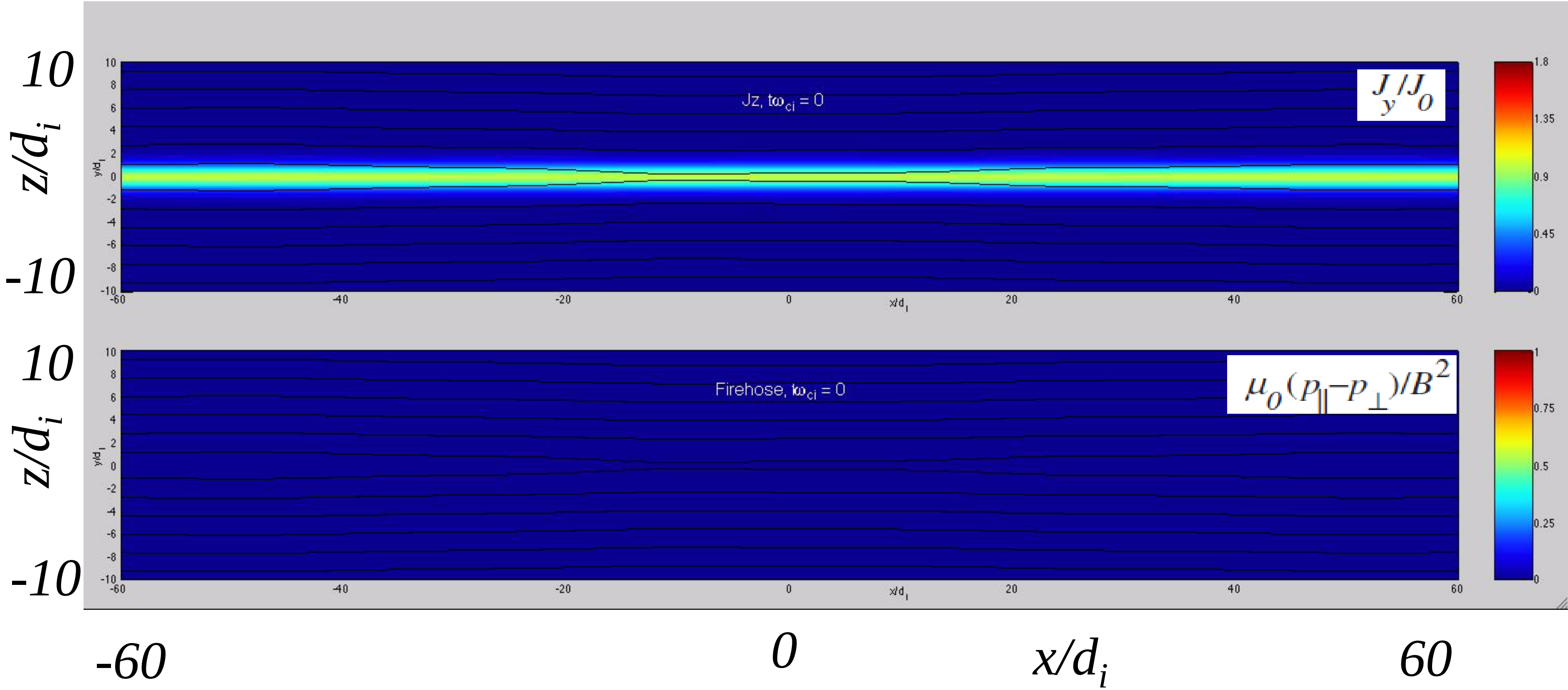
Kinetic simulation results at m_i / m_e = 1836, [A Le et al., PRL 2013]





New Fluid Closure Allows for Exploration of Large Systems

$m_i / m_e = 400$, $B_g / B_0 = 0.28$



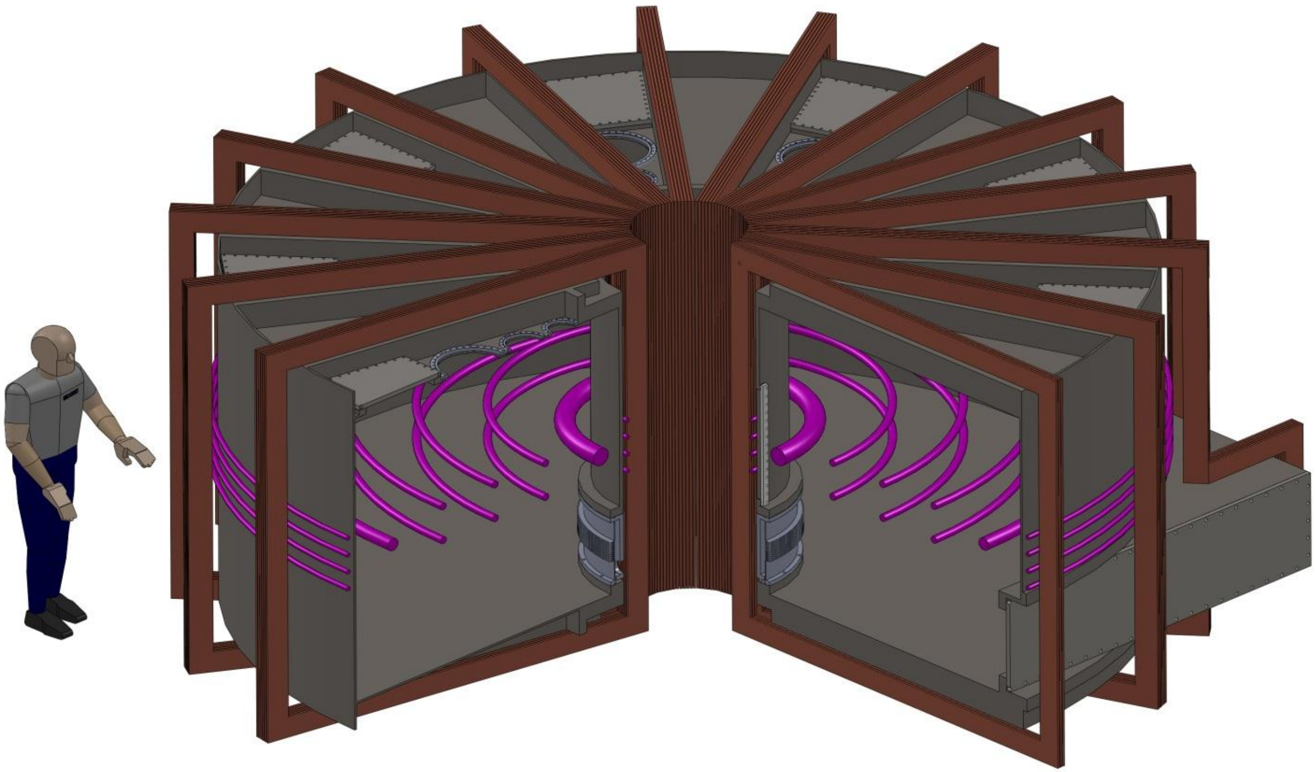
Requirements on New Experiment

- Large normalized size of experiment: $L/d_i \sim 10$ (high n, large L)
- Low collisionallity to allow $p_\parallel >> p_\perp$: $\tau_{ei} v_A > d_i$ (low n, high T_e, high B)
- Low electron pressure: $\beta_e < 0.05$ (low n, T_e, high B)
- Manageable loop voltage: $0.1 v_A B_{rec} (2\pi R) < 5\text{kV}$ (high n, low B)
- Variable guide field: $B_g = 0 - 4 B_{rec}$
- Symmetric inflows

Experimental window available in Hydrogen or Helium plasma with

$n \sim 10^{18}\ \text{m}^{-3}$, $T_e \sim 15$ eV, $B_{rec} \sim 15$ mT, $L \sim 2$ m

New Experiment Needed

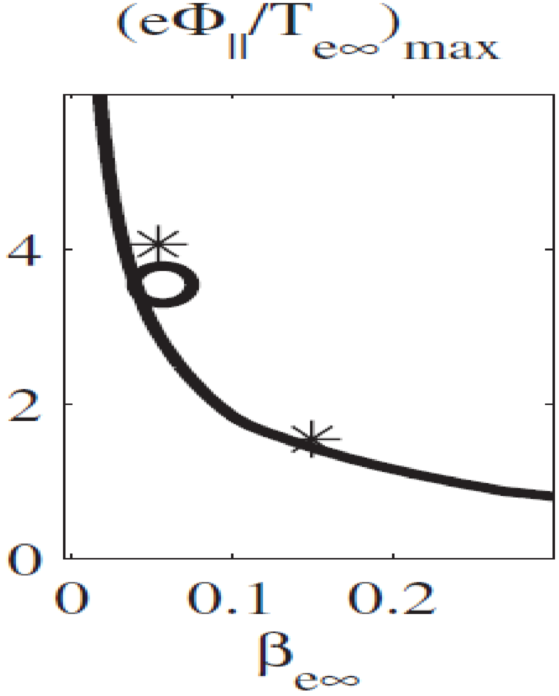


Experimental window available in Hydrogen or Helium plasma with

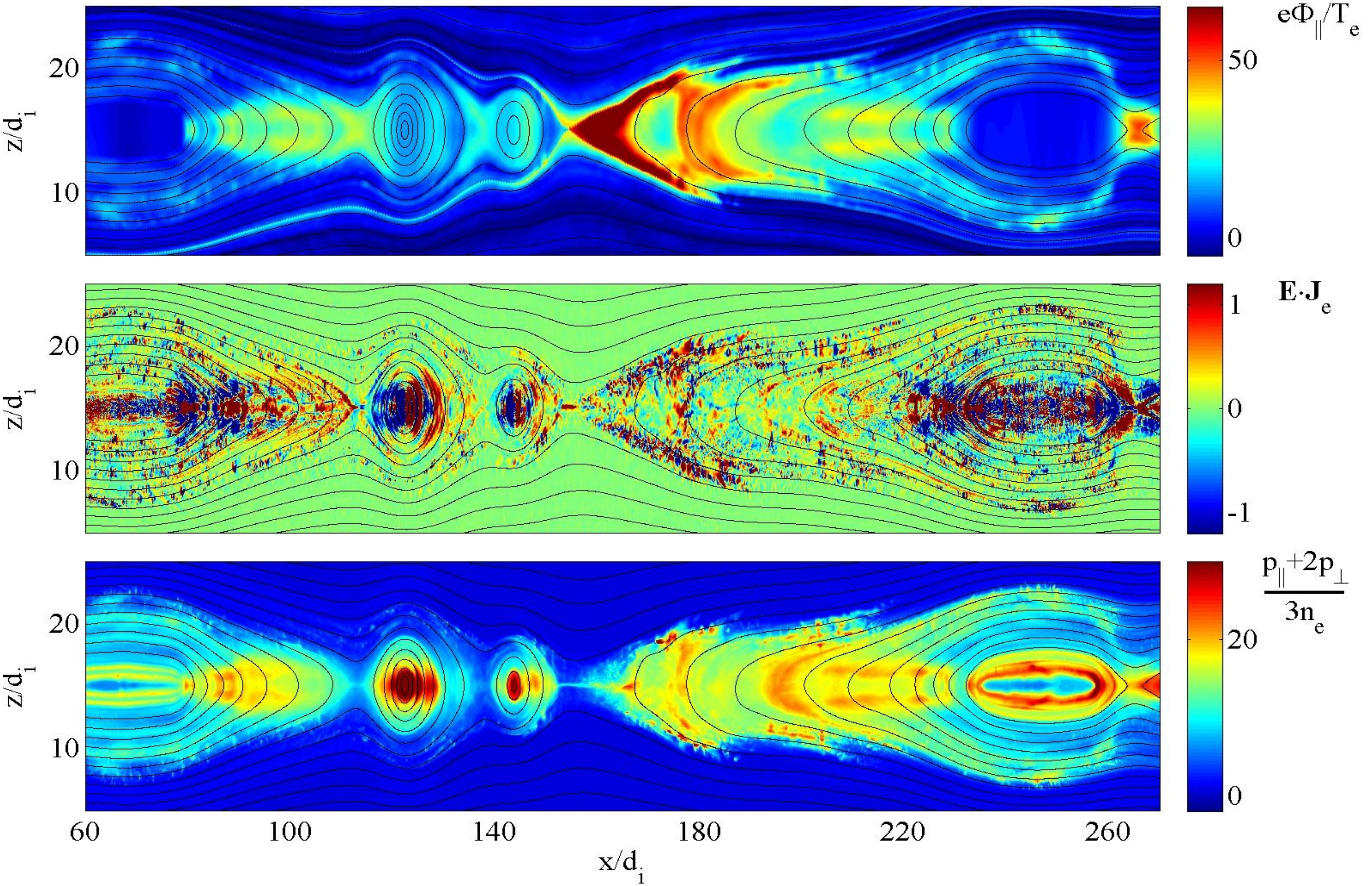
$n \sim 10^{18}$ m^{-3}, $T_e \sim 15$ eV, $B_{rec} \sim 15$ mT, $L \sim 2$ m

Electron Heating

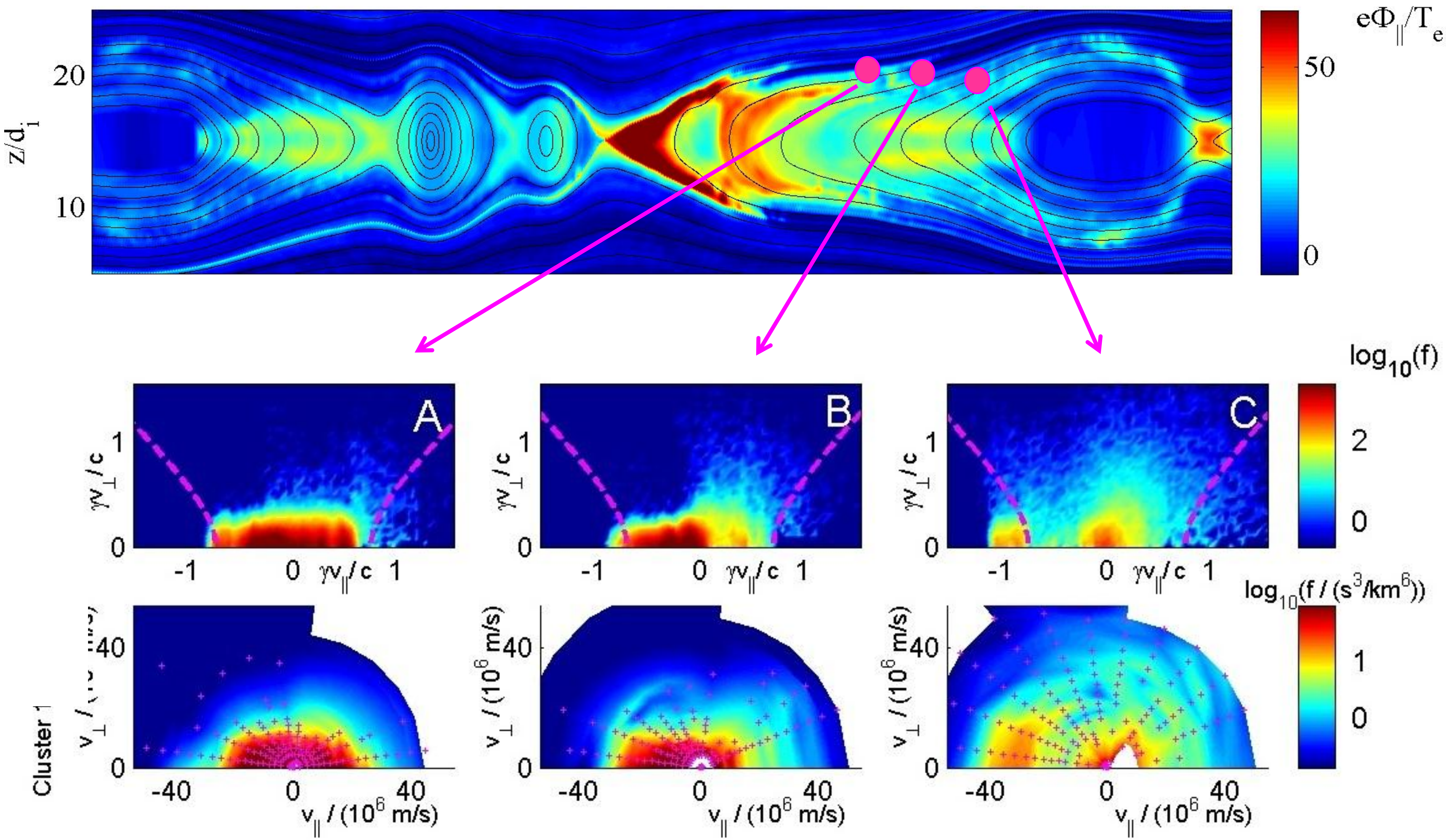
$$\beta_{e\infty} = \frac{\text{plasma pressure}}{\text{magnetic pressure}}$$



New Simulation with $\beta_e \sim 0.003$



Spacecraft Distributions Reproduced



J. Egedal et al., Nature Physics (2012)

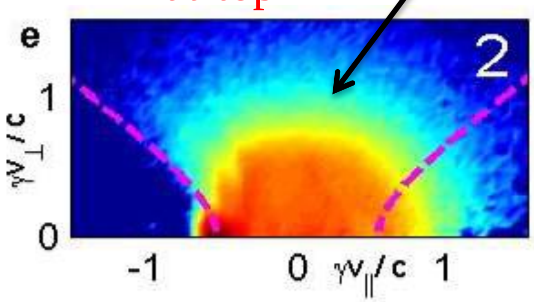
$\Phi_{\parallel}$ confines electrons, allowing sustained energization by $E_{\perp}$

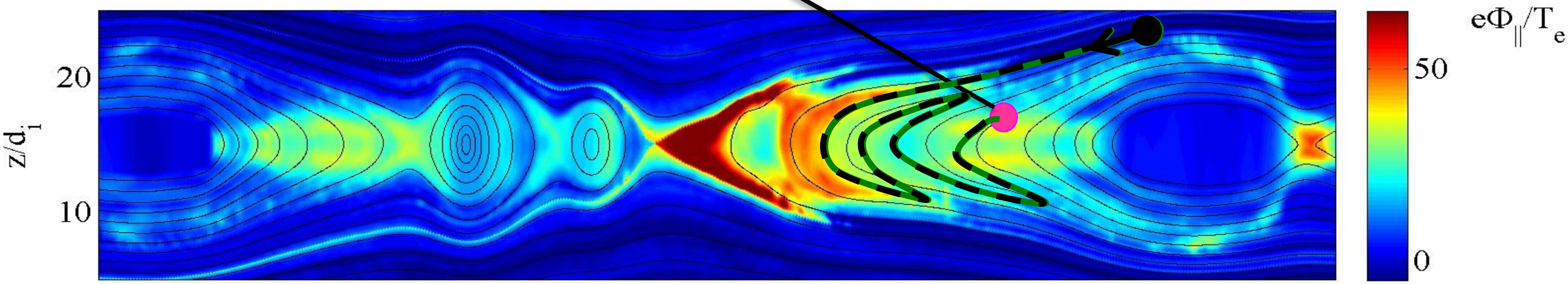
Trapped region with
pitch angle scattering

Heated by $\mathbf{v}_{\mathrm{d}} \cdot \mathbf{E}_{\perp}$,

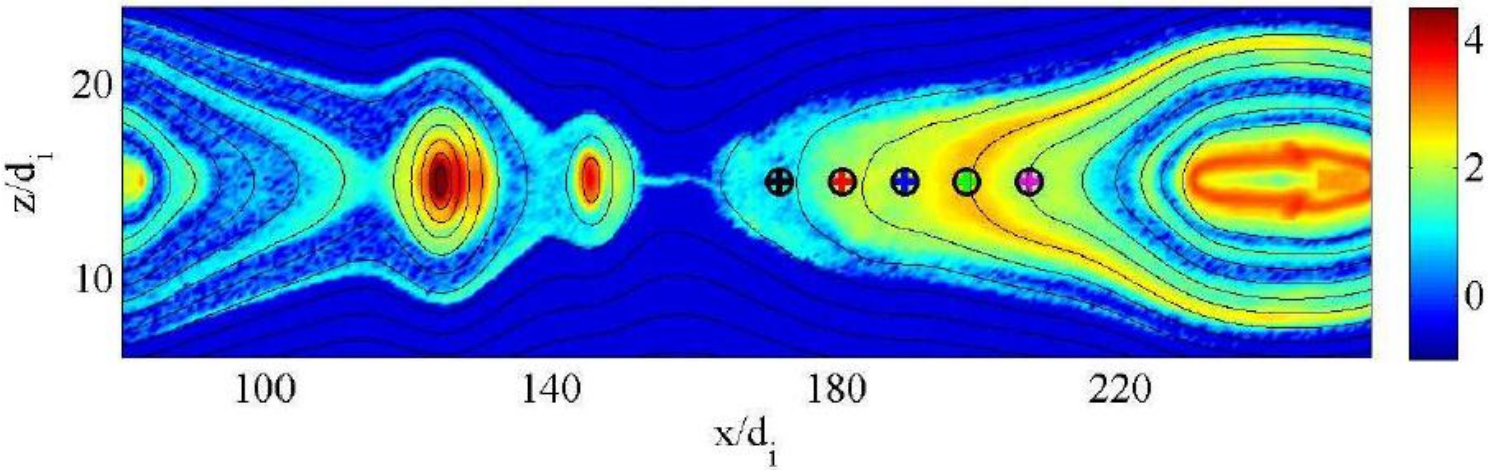
$$\frac{\partial v}{\partial t} \simeq \frac{v_A}{\tau_b}$$

Flat-top



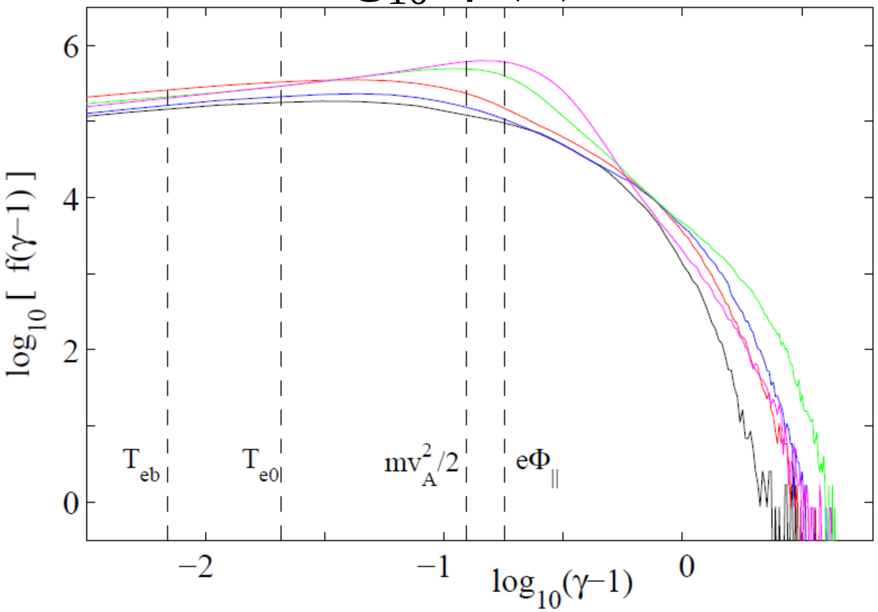


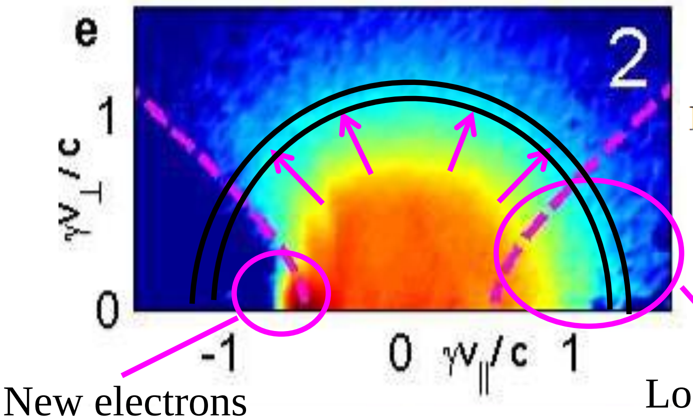
Generation of Super-Thermals



$\log_{10}$ (# of electrons with energies above $1.7mc^2$)

$\log_{10}[\, f\,(E)\,]$





$$\Gamma_v \;=\; f\, 4\pi v^2 \frac{\partial v}{\partial t}$$

$$\frac{\partial v}{\partial t} \simeq \frac{v_A}{\tau_b}$$

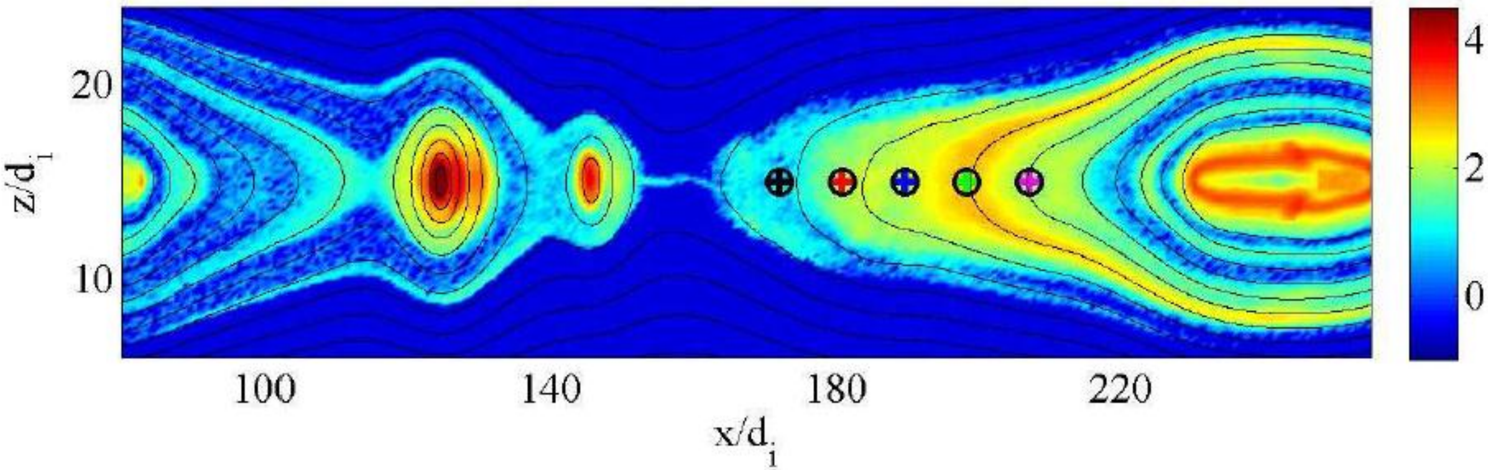
Lost electrons

$$\Gamma_l = dv\, f\, \frac{4\pi v^2}{\tau_b} R_{\rm loss}$$

Steady state:

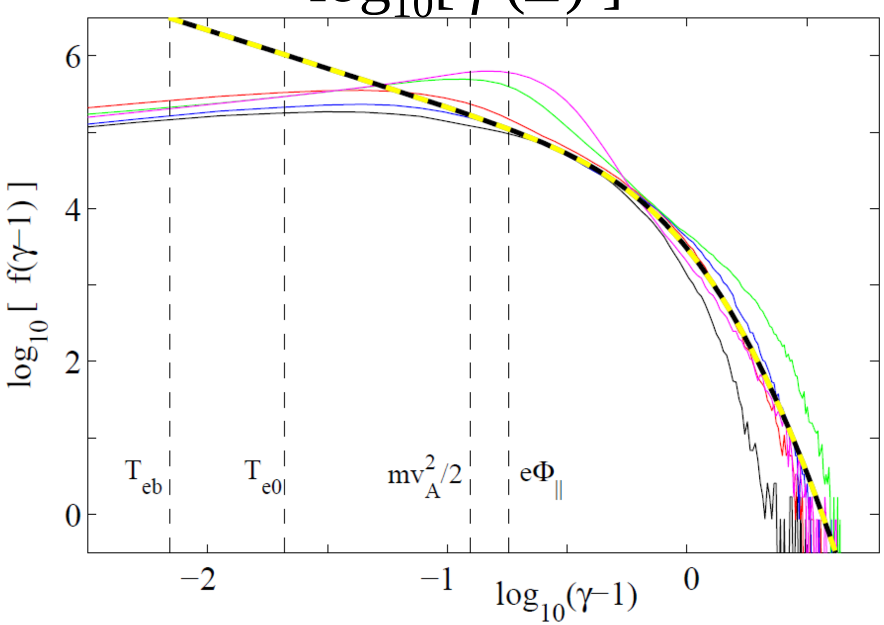
$$dv \frac{d\Gamma_v}{dv} \;=\; \Gamma_l \;\rightarrow\; f \;=\; \frac{A}{v^3} \exp\left(-\frac{v\, R_{\rm loss}}{v_A} \right)$$

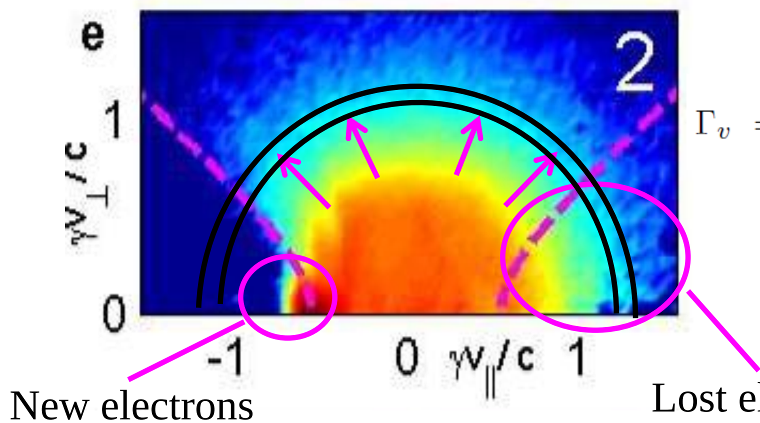
Generation of Super-Thermals



$\log_{10}$ (# of electrons with energies above 1.7mc^2)

$\log_{10}[\, f\,(E)\ ]$





$$\Gamma_v \;=\; f\,4\pi v^2 \frac{\partial v}{\partial t}$$

$$\frac{\partial v}{\partial t} \simeq \frac{v_A}{\tau_b}$$

$$\Gamma_l = dv\, f\, \frac{4\pi v^2}{\tau_b} R_{\mathrm{loss}}$$

Steady state:

$$dv \frac{d\Gamma_v}{dv} \;=\; \Gamma_l \;\rightarrow\; f \;=\; \frac{A}{v^3} \exp\left(-\frac{v\, R_{\mathrm{loss}}}{v_A}\right)$$

Flare Heating by Parallel E-fields?

Reconnection in flares occurs where $\beta_{e\infty} < 0.01$

Before reconnection: $p = nT_e$

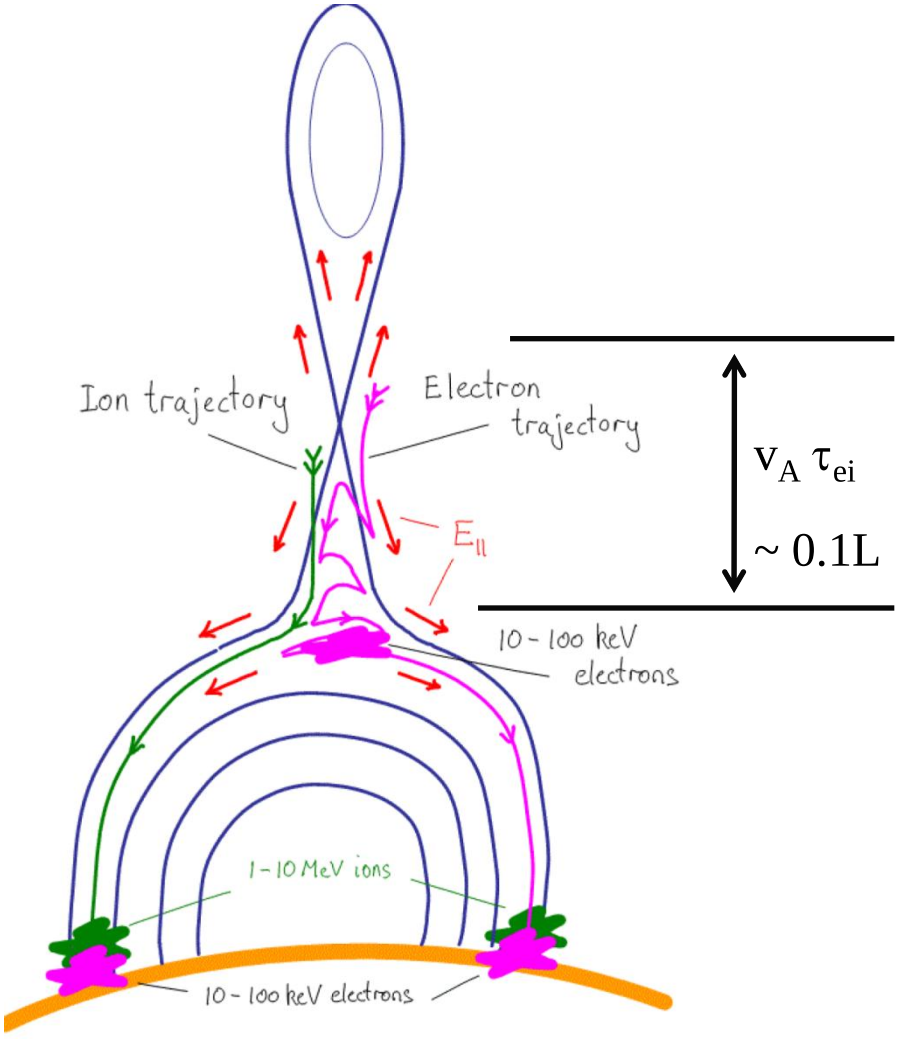
➔ $e\Phi_{\|} \sim T_e \log(n/n_0)$

During reconnection:

Electrons are collisionless

Observations need

$e\Phi_{\|} \approx 200T_e$



Conclusions

- Inspired by experimental and spacecraft observations, a new model for pressure anisotropy is derived. It includes the effects of trapped electrons as the main driver of the anisotropy.

- The results from the model have been confirmed through comprehensive numerical simulations. It provides quantative insight to how the pressure anisotropy governs the structure of the reconnection region.