(Some) effects of a weak magnetic field on plasma dynamics

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A survey of anisotropic diffusive (large-scale) instabilities

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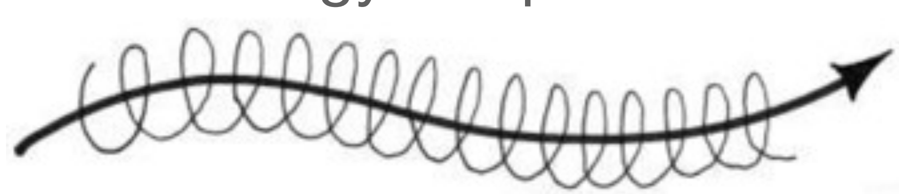
- $\hat{\boldsymbol{b}}\hat{\boldsymbol{b}}:\nabla\boldsymbol{v}$: Anisotropic (Braginskii) viscosity and rotational stability

Galactic centre at Bondi radius

$$\ell \sim 0.1 \text{ pc}$$
$$\lambda_{\rm mfp} \sim 0.01 \text{ pc}$$
$$\rho_{\rm i} \sim 1 \text{ ppc}$$

$$t_{\rm dyn} \sim 200 \text{ yr}$$
$$t_{\rm ii,coll} \sim 20 \text{ yr}$$
$$t_{\rm gyr,i} \sim 0.1 \text{ s}$$

gyrotropic

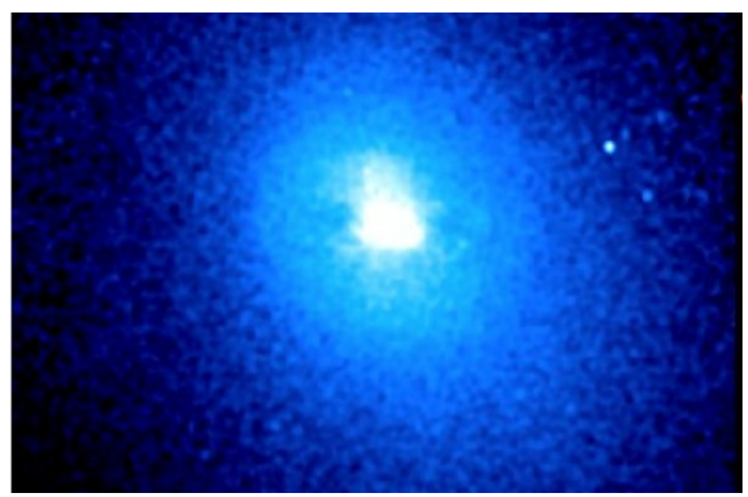


A survey of anisotropic diffusive (large-scale) instabilities

- $\hat{\boldsymbol{b}}\hat{\boldsymbol{b}}:\nabla \boldsymbol{v}$: Anisotropic (Braginskii) viscosity and rotational stability
- $\hat{\boldsymbol{b}}\hat{\boldsymbol{b}}\cdot\nabla T$: Anisotropic conduction and convective stability

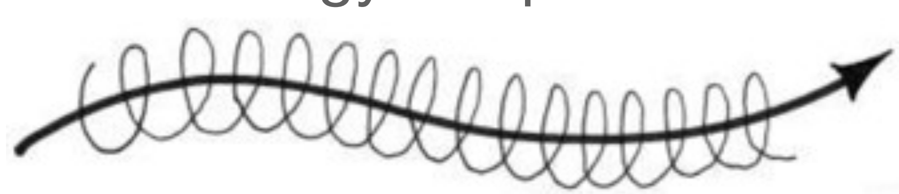
intracluster medium of galaxy clusters

$$\ell \sim 100 \text{ kpc}$$
$$\lambda_{\text{mfp}} \sim 1 \text{ kpc}$$
$$\rho_{\text{i}} \sim 1 \text{ npc}$$



$$t_{\text{dyn}} \gtrsim 100 \text{ Myr}$$
$$t_{\text{ii,coll}} \sim 1 - 10 \text{ Myr}$$
$$t_{\text{gyr,i}} \sim 10 \text{ min}$$

gyrotropic



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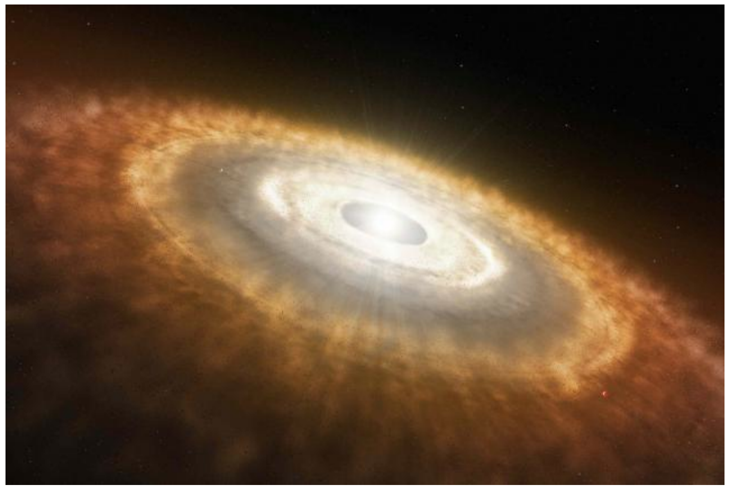
- $\hat{\boldsymbol{b}}\hat{\boldsymbol{b}}\cdot\nabla T$: Anisotropic conduction and convective stability
- $(\hat{\boldsymbol{k}}\times\hat{\boldsymbol{b}})(\hat{\boldsymbol{k}}\times\hat{\boldsymbol{b}}):\nabla\boldsymbol{v}$: Ambipolar diffusion and shear stability
- $(\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{b}})\ \hat{\boldsymbol{k}}\cdot(\nabla\times\boldsymbol{v})$: Hall effect and vortex stability

protoplanetary disk at 1 AU

$$\ell \sim 1\ \mathrm{AU}$$
$$r_{\mathrm{gyr,i}} \sim 40\ \mathrm{m}$$
$$\lambda_{\mathrm{mfp,i}} \sim 0.1\ \mathrm{cm}$$



$$t_{\mathrm{ni,coll}} \sim 1\ \mathrm{Myr}$$
$$t_{\mathrm{dyn}} \sim 1\ \mathrm{yr}$$
$$t_{\mathrm{gyr,i}} \sim 40\ \mathrm{ms}$$
$$t_{\mathrm{in,coll}} \sim 3\ \mu\mathrm{s}$$

A survey of anisotropic diffusive (large-scale) instabilities

- $\hat{\boldsymbol{b}}\hat{\boldsymbol{b}} : \nabla \boldsymbol{v}$: Anisotropic (Braginskii) viscosity and rotational stability
- $\hat{\boldsymbol{b}}\hat{\boldsymbol{b}} \cdot \nabla T$: Anisotropic conduction and convective stability
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All of these instabilities can be understood without magnetic tension:

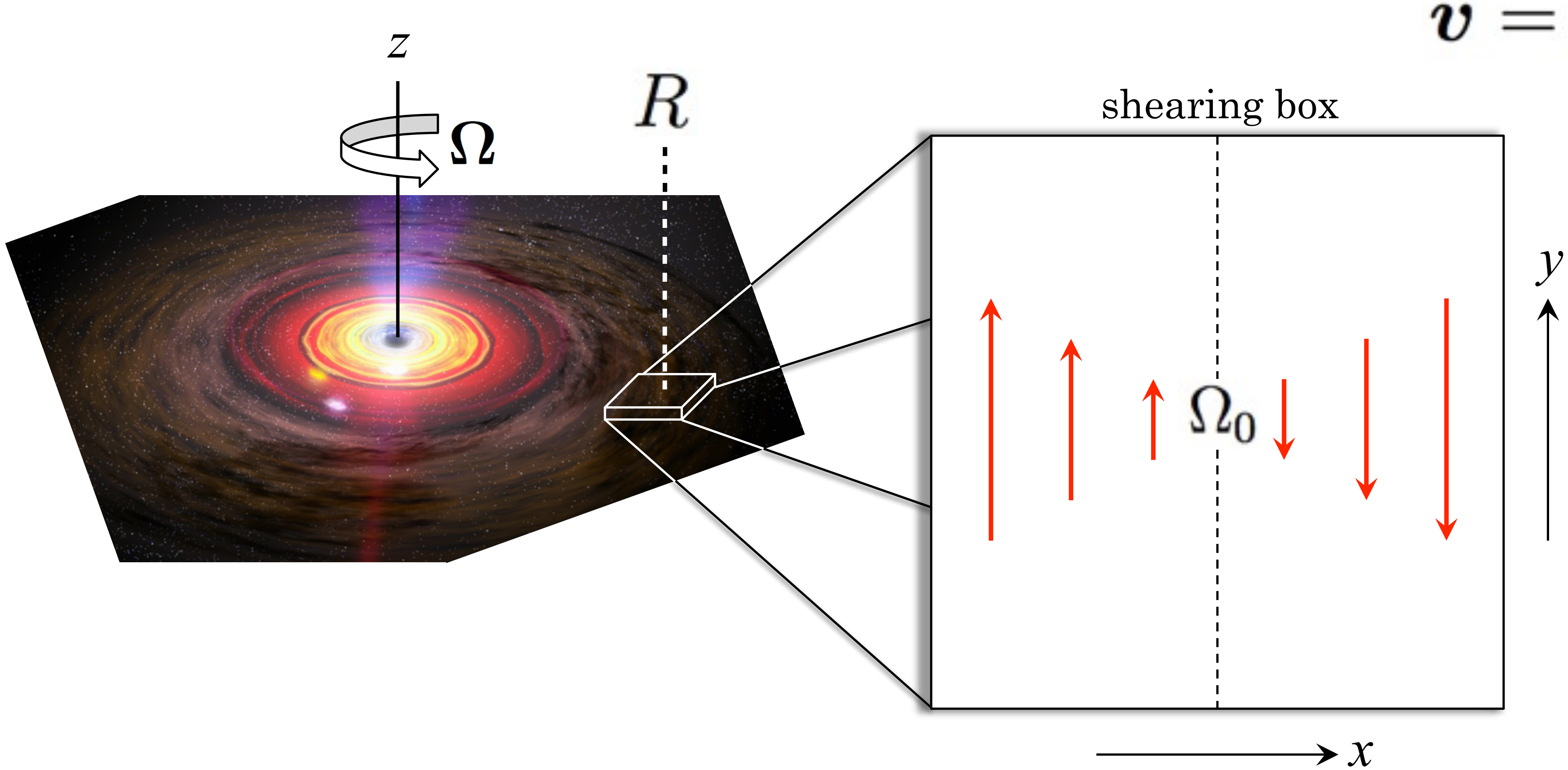
$$\cancel{(\boldsymbol{k} \cdot \boldsymbol{v}_{\mathbf{A}})^2}$$

First, some notation...

- Lagrangian displacement: $\boldsymbol{\xi}$
- Lagrangian perturbation: $\Delta = \delta + \boldsymbol{\xi}\boldsymbol{\cdot}\boldsymbol{\nabla}$
- Growth rate: γ
- Wavenumber: $\boldsymbol{k}$
- Magnetic-field unit vector: $\hat{\boldsymbol{b}}$

Anisotropic viscosity and rotational stability

$$\boldsymbol{v} = x\frac{\mathrm{d}\Omega}{\mathrm{d}\ln R}\,\hat{\boldsymbol{y}}$$



Anisotropic viscosity and rotational stability

We start with the force equation, in the limit of fast viscosity acting on Eulerian perturbations:

$$\nabla \cdot \mathbf{P} \simeq 0$$

with $\mathbf{P} = \hat{\boldsymbol{b}}\hat{\boldsymbol{b}}\,(\delta p_{||} - \delta p_{\perp}) = -\nu\,\hat{\boldsymbol{b}}\hat{\boldsymbol{b}}\left(\hat{\boldsymbol{b}}\hat{\boldsymbol{b}}\boldsymbol{:}\nabla\delta\boldsymbol{v} + \delta\hat{\boldsymbol{b}}\,\hat{\boldsymbol{b}}\boldsymbol{:}\nabla\boldsymbol{v} + \hat{\boldsymbol{b}}\,\delta\hat{\boldsymbol{b}}\boldsymbol{:}\nabla\boldsymbol{v}\right)$

One can show from this that, in (most) equilibrium rotating disks,

$$\Delta\Omega \simeq 0$$

(magnetically connected) fluid elements maintain their angular velocity as they are displaced.

For sufficiently weak collisions, field lines become isotachs.

Anisotropic viscosity and rotational stability

For sufficiently weak collisions, field lines become isotachs.

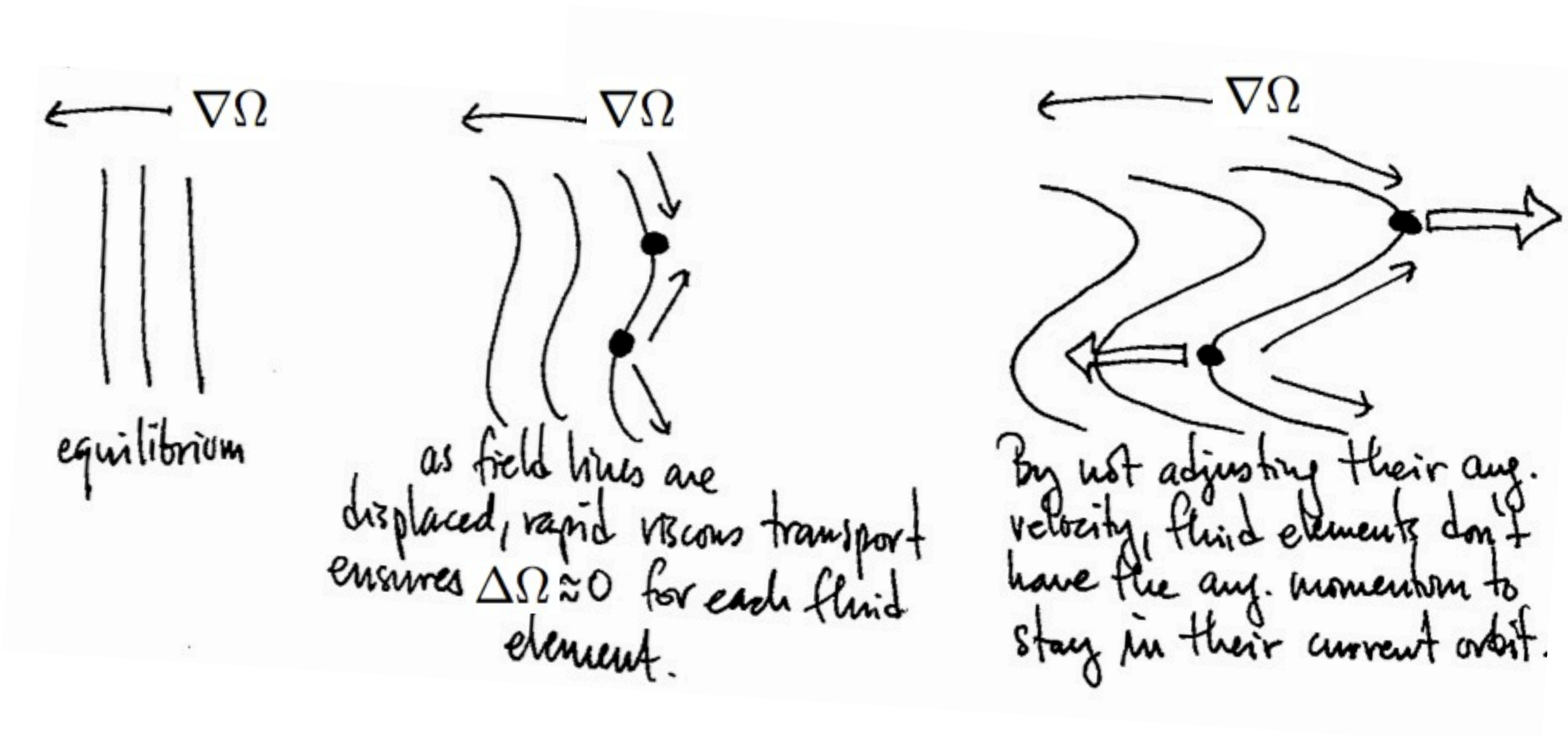
This means that the change in a fluid element's angular momentum is

$$\frac{\Delta L}{L_0} = 2\frac{\xi_x}{R} + \frac{\Delta\Omega}{\Omega_0} \simeq 2\frac{\xi_x}{R}$$

Outwardly (inwardly) displaced fluid elements gain (lose) angular momentum.

For $\dfrac{\mathrm{d}\Omega}{\mathrm{d}\ln R} < 0$, this is enough to guarantee instability.

Magnetoviscous instability



Dispersion relation (Balbus 2004)

$$\gamma^2\left(\gamma^2+\gamma\,\omega_{\rm visc}\frac{k_\perp^2}{k^2}+g\frac{\mathrm{d}\ln\Omega^2}{\mathrm{d}R}\frac{k_z^2}{k^2}\right)=-\gamma\omega_{\rm visc}\,g\frac{\mathrm{d}\ln\Omega^2}{\mathrm{d}R}\frac{k_z^2 b_y^2}{k^2}-4\Omega^2\gamma^2\frac{k_z^2}{k^2}$$

Alfvén wave

slow mode

anisotropic viscous damping

free-energy gradient (shear)

viscous coupling of slow & Alfvén modes

epicyclic coupling

$$\gamma_{\rm max}\simeq\left(-g\frac{\mathrm{d}\ln\Omega^2}{\mathrm{d}R}\right)^{1/2}$$

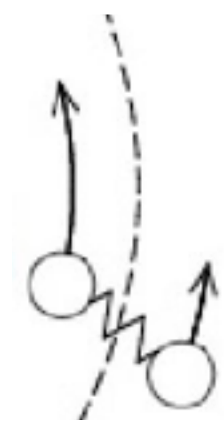
$$\boldsymbol{g}\cdot\boldsymbol{\nabla}\ln L^2<0\longrightarrow\boldsymbol{g}\cdot\boldsymbol{\nabla}\ln\Omega^2<0$$ for stability

Collisionless MRI (Quataert, Dorland & Hammett 2002)

This behavior is not unique to the Braginskii closure. In fact, the details of the closure don't matter much (one can obtain this instability with CGL equations, or full kinetic equations, or kinetic MHD with Landau closure).

$$\ddot{\xi}_x - 2\Omega\dot{\xi}_y = -\frac{\mathrm{d}\Omega^2}{\mathrm{d}\ln R}\,\xi_x$$

$$\ddot{\xi}_y + 2\Omega\dot{\xi}_x = -K\xi_y$$

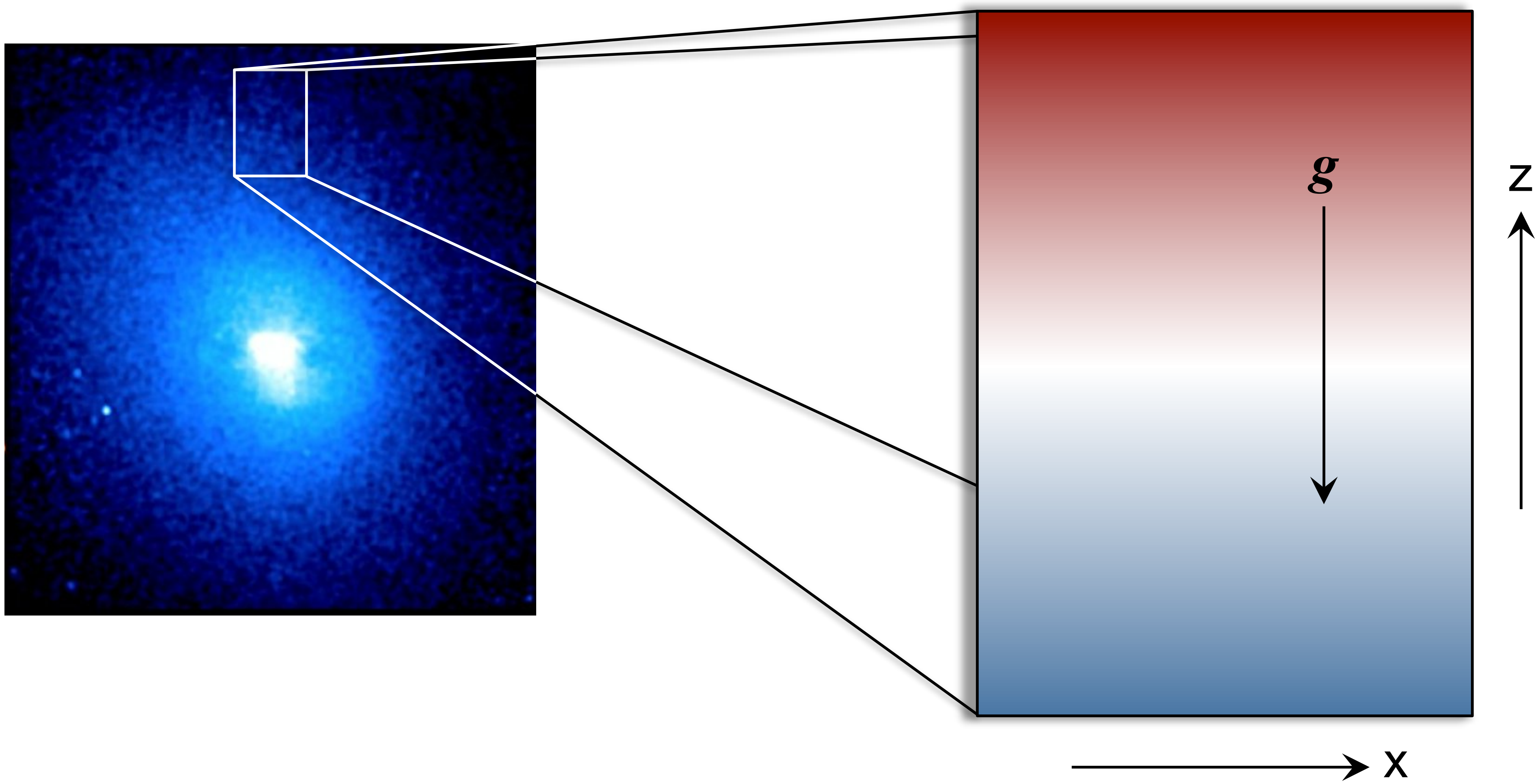


spring only operates in y direction

$$\rightarrow\quad \gamma^2\left(\gamma^2 + K + g\frac{\mathrm{d}\ln\Omega^2}{\mathrm{d}R}\right) = -K\,g\frac{\mathrm{d}\ln\Omega^2}{\mathrm{d}R} - 4\Omega^2\gamma^2$$

$$\gamma^2\left(\gamma^2 + \gamma\,\omega_{\mathrm{visc}}\frac{k_\perp^2}{k^2} + g\frac{\mathrm{d}\ln\Omega^2}{\mathrm{d}R}\frac{k_z^2}{k^2}\right) = -\gamma\omega_{\mathrm{visc}}\,g\frac{\mathrm{d}\ln\Omega^2}{\mathrm{d}R}\frac{k_z^2 b_y^2}{k^2} - 4\Omega^2\gamma^2\frac{k_z^2}{k^2}$$

Anisotropic conduction and convective stability

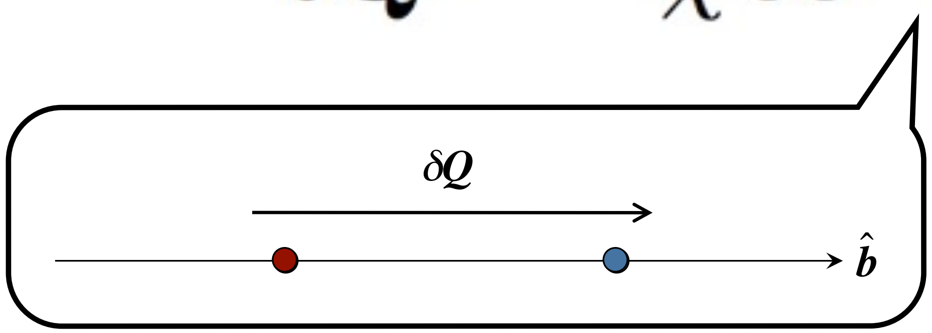


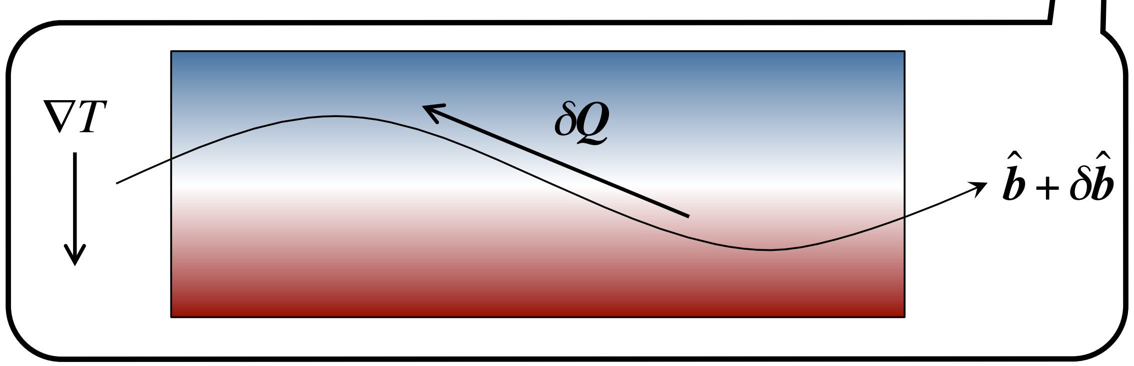
Anisotropic conduction and convective stability

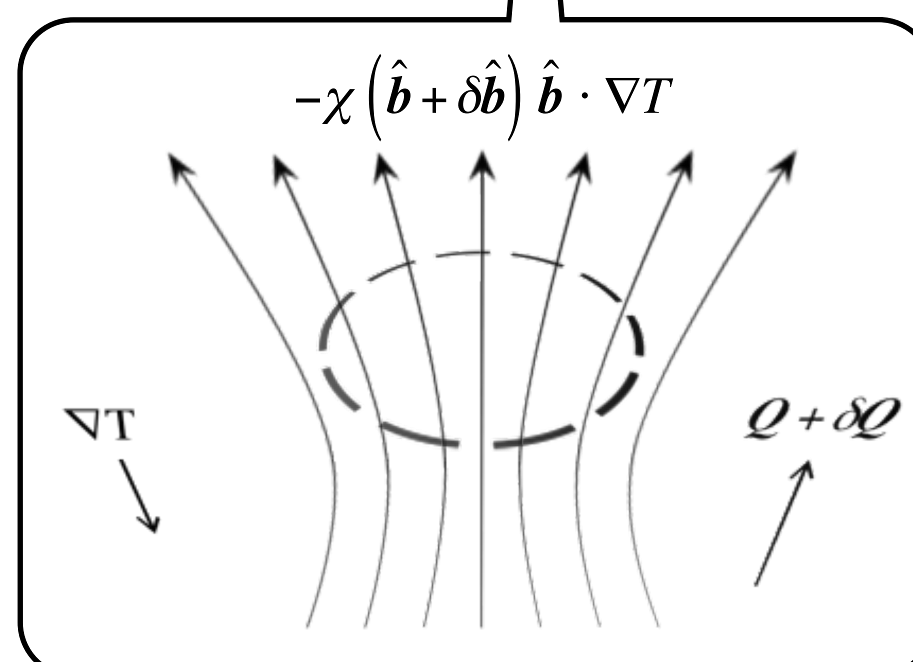
We start with the entropy equation, in the limit of fast conduction acting on Eulerian perturbations:

$$\nabla \cdot \delta \boldsymbol{Q} \simeq 0$$

with $\delta \boldsymbol{Q} = -\chi \, \hat{\boldsymbol{b}}\hat{\boldsymbol{b}} \cdot \nabla \delta T - \chi \, \hat{\boldsymbol{b}} \, \delta\hat{\boldsymbol{b}} \cdot \nabla T - \chi \, \delta\hat{\boldsymbol{b}} \, \hat{\boldsymbol{b}} \cdot \nabla T$







Anisotropic conduction and convective stability

$$\nabla \cdot \left(\delta \boldsymbol{Q} = -\chi \, \hat{\boldsymbol{b}}\hat{\boldsymbol{b}} \cdot \nabla \delta T - \chi \, \hat{\boldsymbol{b}} \, \delta\hat{\boldsymbol{b}} \cdot \nabla T - \chi \, \delta\hat{\boldsymbol{b}} \, \hat{\boldsymbol{b}} \cdot \nabla T \right)$$

Eulerian to Lagrangian translation

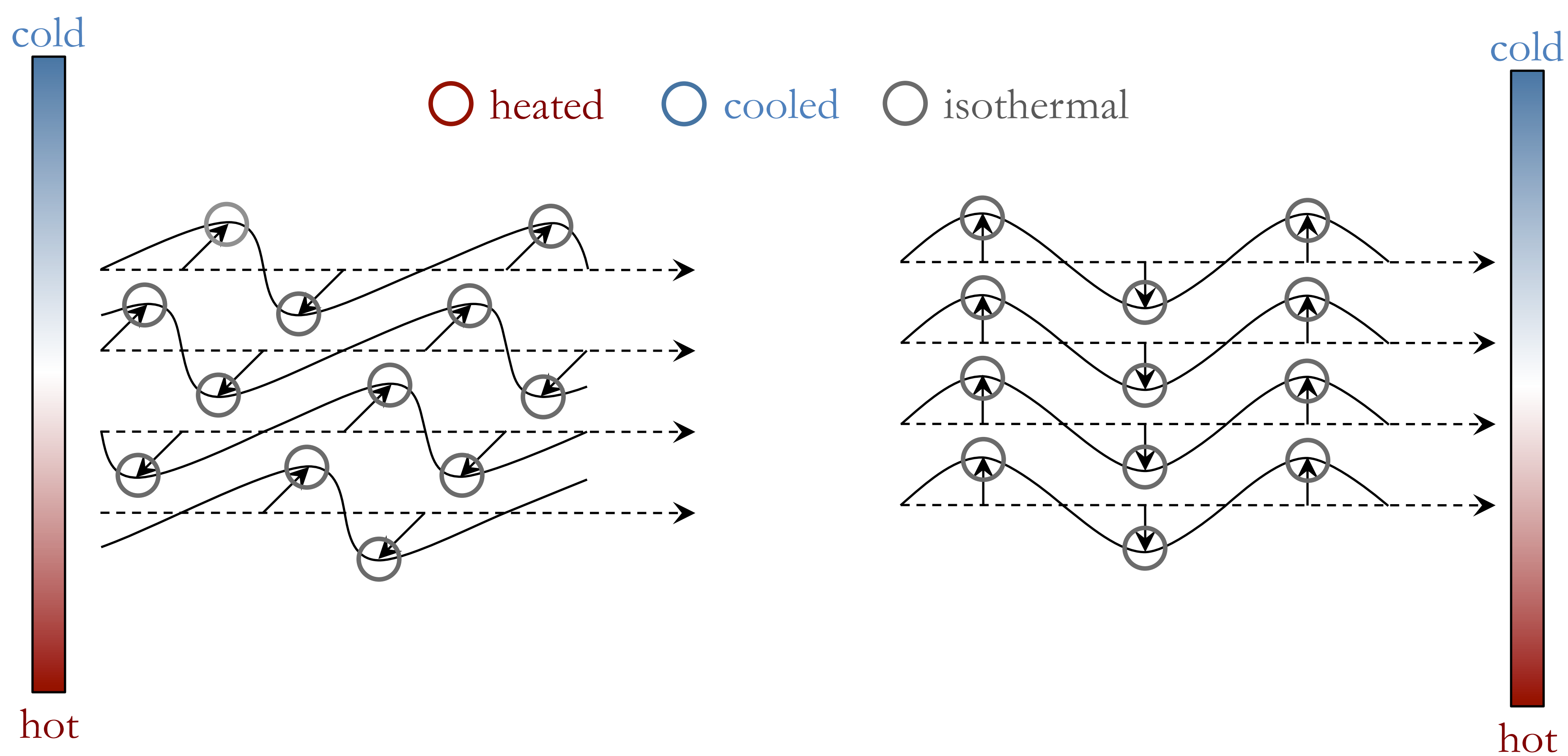
$$-k_{||}^2 \left(\Delta T - \boldsymbol{\xi} \cdot \nabla T\right) \qquad - k_{||}^2 \, \boldsymbol{\xi}_{\perp} \cdot \nabla_{\perp} T \qquad + k_{||}^2 \, \xi_{||} \nabla_{||} T$$

$$\longrightarrow \Delta T \simeq 2 \xi_{||} \nabla_{||} T$$

Anisotropic conduction and convective stability

$$\Delta T \simeq 2\xi_{||}\nabla_{||}T$$

i.e. compressions/rarefactions in $\boldsymbol{\nabla} T$ -oriented field lines lead to heating/cooling



Anisotropic conduction and convective stability

$$\Delta T \simeq 2\xi_{||}\nabla_{||}T$$

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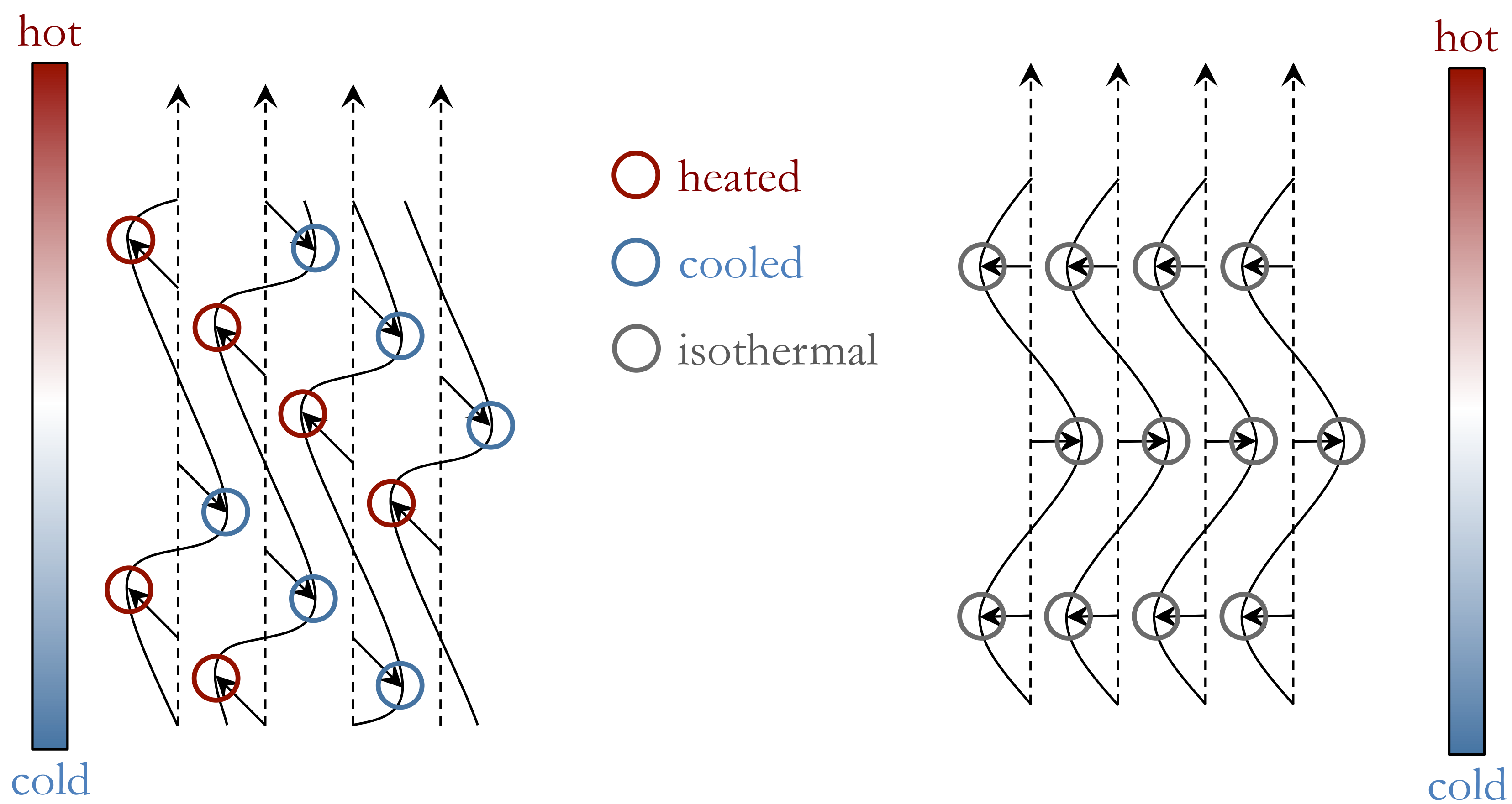

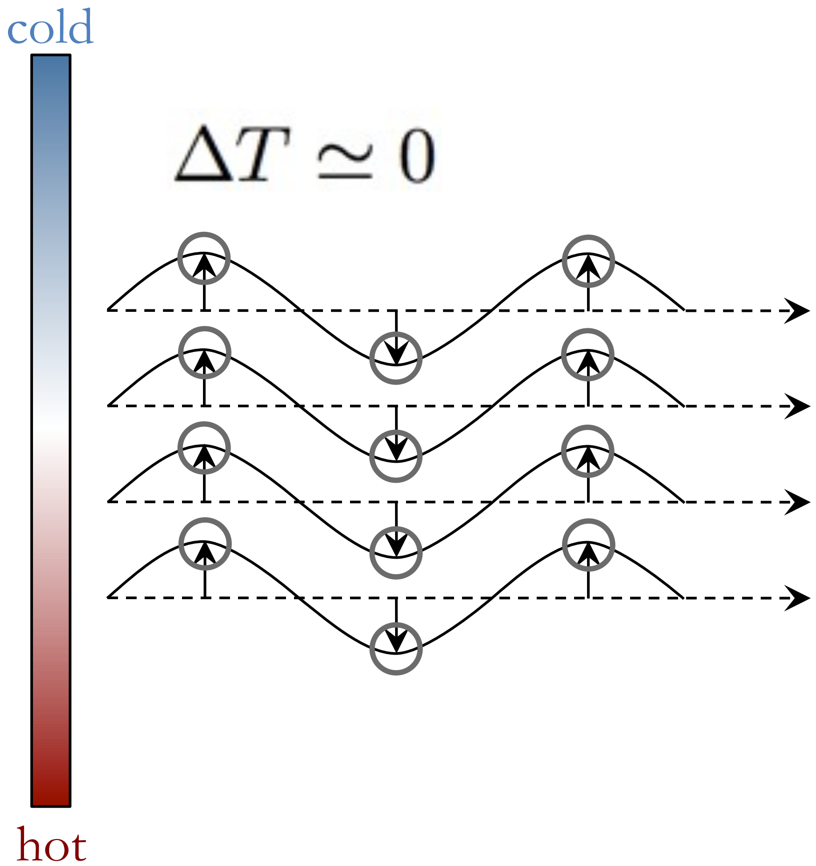

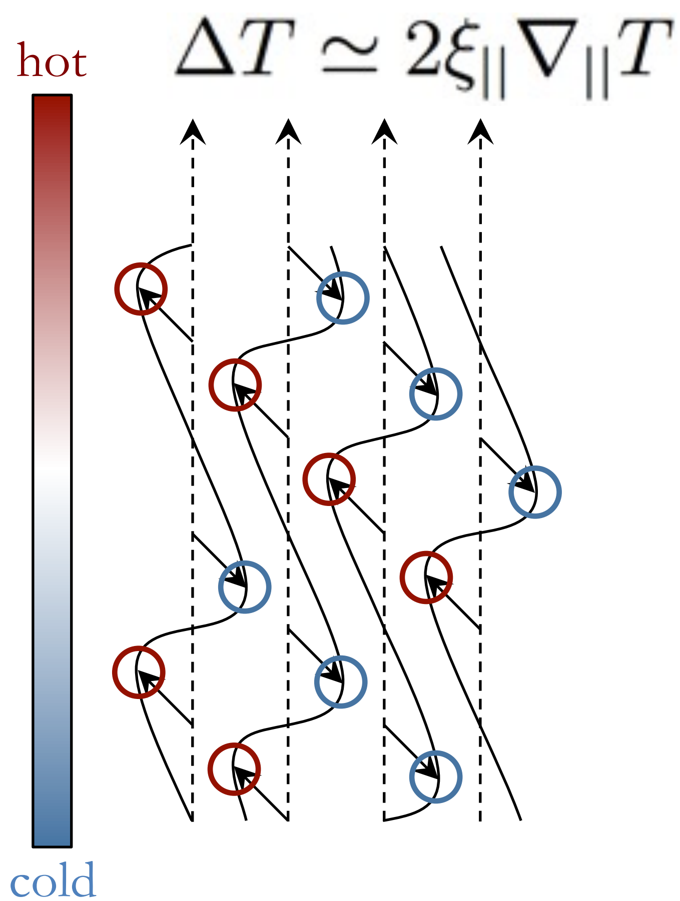

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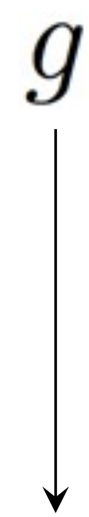

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Magneto-Thermal Instability

(Balbus 2000, 2001)

Heat-flux-driven Buoyancy Instability

(Quataert 2008)

Dispersion relation (Kunz 2011)

$$\gamma^2 \left(\gamma^2 + \gamma\, \omega_{\rm visc} \frac{k_\perp^2}{k^2} + g \frac{\mathrm{d} \ln T}{\mathrm{d}z} \frac{\mathcal{K}}{k^2} \right) \simeq -\gamma \omega_{\rm visc}\, g \frac{\mathrm{d} \ln T}{\mathrm{d}z} \frac{b_x^2 k_y^2}{k^2}$$

- γ^2: Alfvén wave
- γ^2 (inside parentheses): slow mode
- $\gamma\, \omega_{\rm visc} \frac{k_\perp^2}{k^2}$: anisotropic viscous damping
- $g \frac{\mathrm{d} \ln T}{\mathrm{d}z} \frac{\mathcal{K}}{k^2}$: altered buoyant response
- right-hand side: viscous coupling of slow & Alfvén modes

$$\gamma_{\rm max} \simeq \left| g \frac{\mathrm{d} \ln T}{\mathrm{d}z} \right|^{1/2}$$

$$\boldsymbol{g} \cdot \boldsymbol{\nabla} \ln P \rho^{-\gamma} < 0 \longrightarrow \boldsymbol{g} \cdot \boldsymbol{\nabla} \ln T = 0$$ for stability

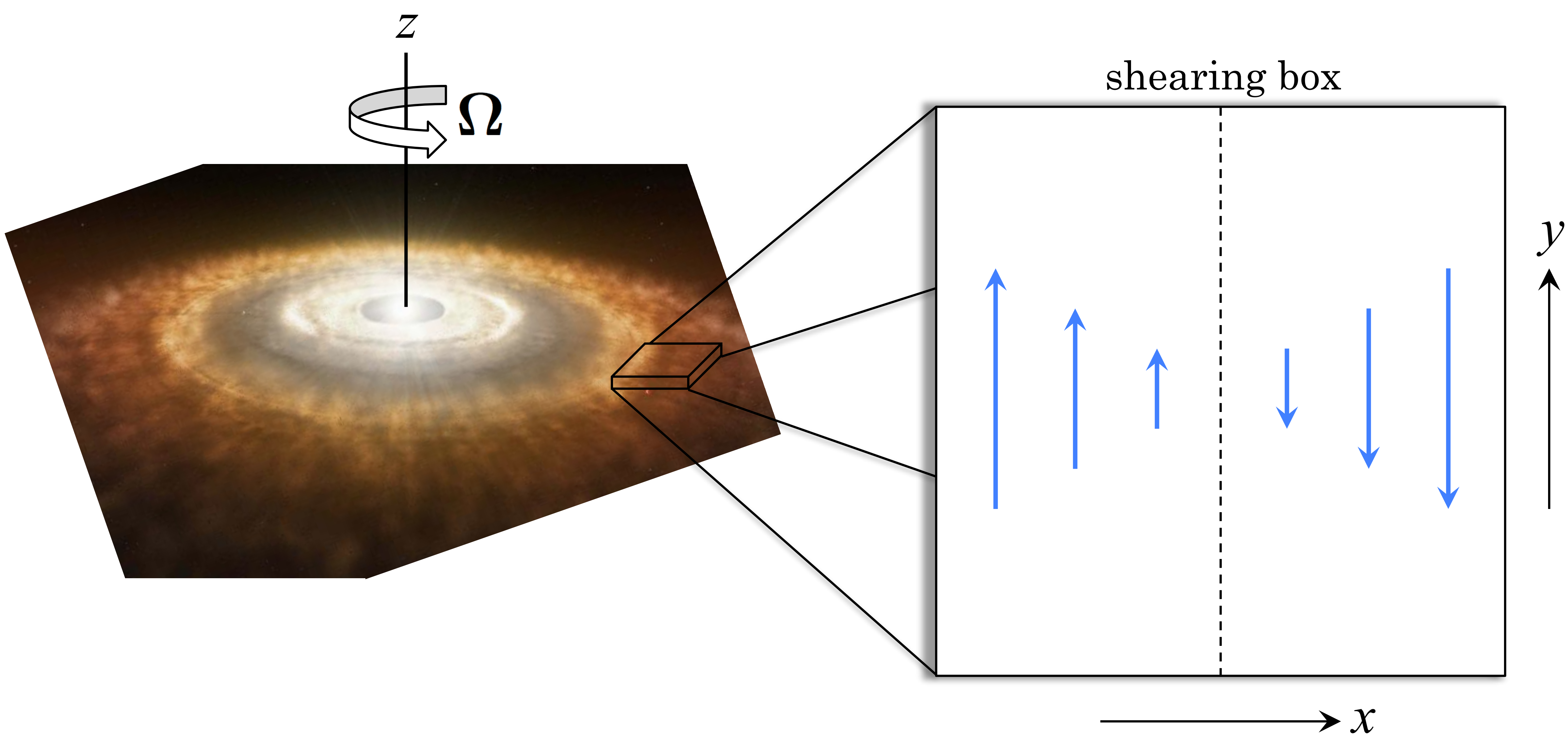
$$\gamma^2\left(\gamma^2+\gamma\,\omega_{\rm visc}\frac{k_\perp^2}{k^2}+g\frac{{\rm d}\ln\Omega^2}{{\rm d}R}\frac{k_z^2}{k^2}\right)=-\gamma\omega_{\rm visc}\,g\frac{{\rm d}\ln\Omega^2}{{\rm d}R}\frac{k_z^2b_y^2}{k^2}-4\Omega^2\gamma^2\frac{k_z^2}{k^2}$$

↓ ↓

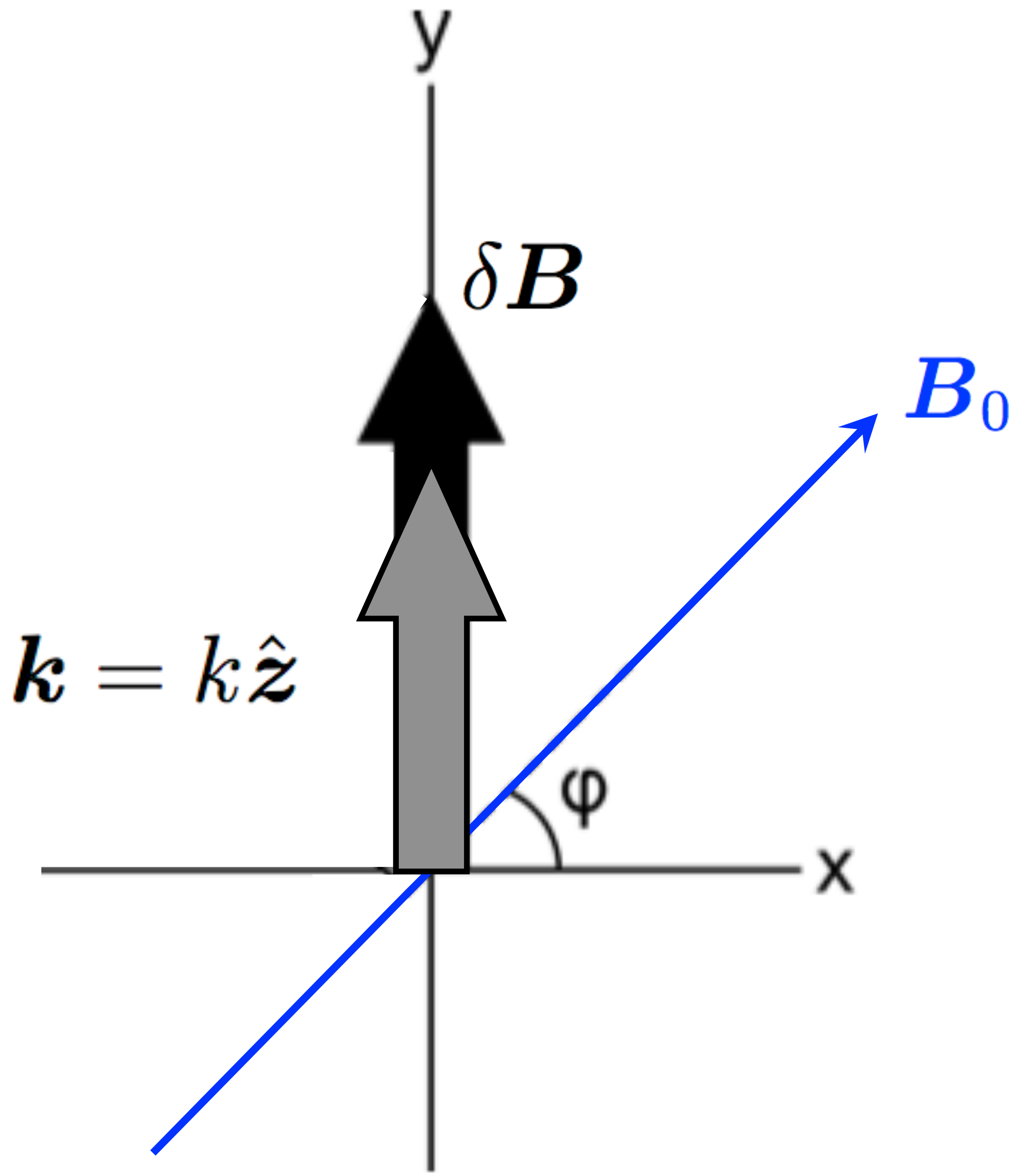
$$\gamma^2\left(\gamma^2+\gamma\,\omega_{\rm visc}\frac{k_\perp^2}{k^2}+g\frac{{\rm d}\ln T}{{\rm d}z}\frac{\mathcal{K}}{k^2}\right)\simeq-\gamma\omega_{\rm visc}\,g\frac{{\rm d}\ln T}{{\rm d}z}\frac{b_x^2k_y^2}{k^2}$$

see Balbus 2000 for more on this analogy

Ambipolar diffusion, Hall effect, and shear stability

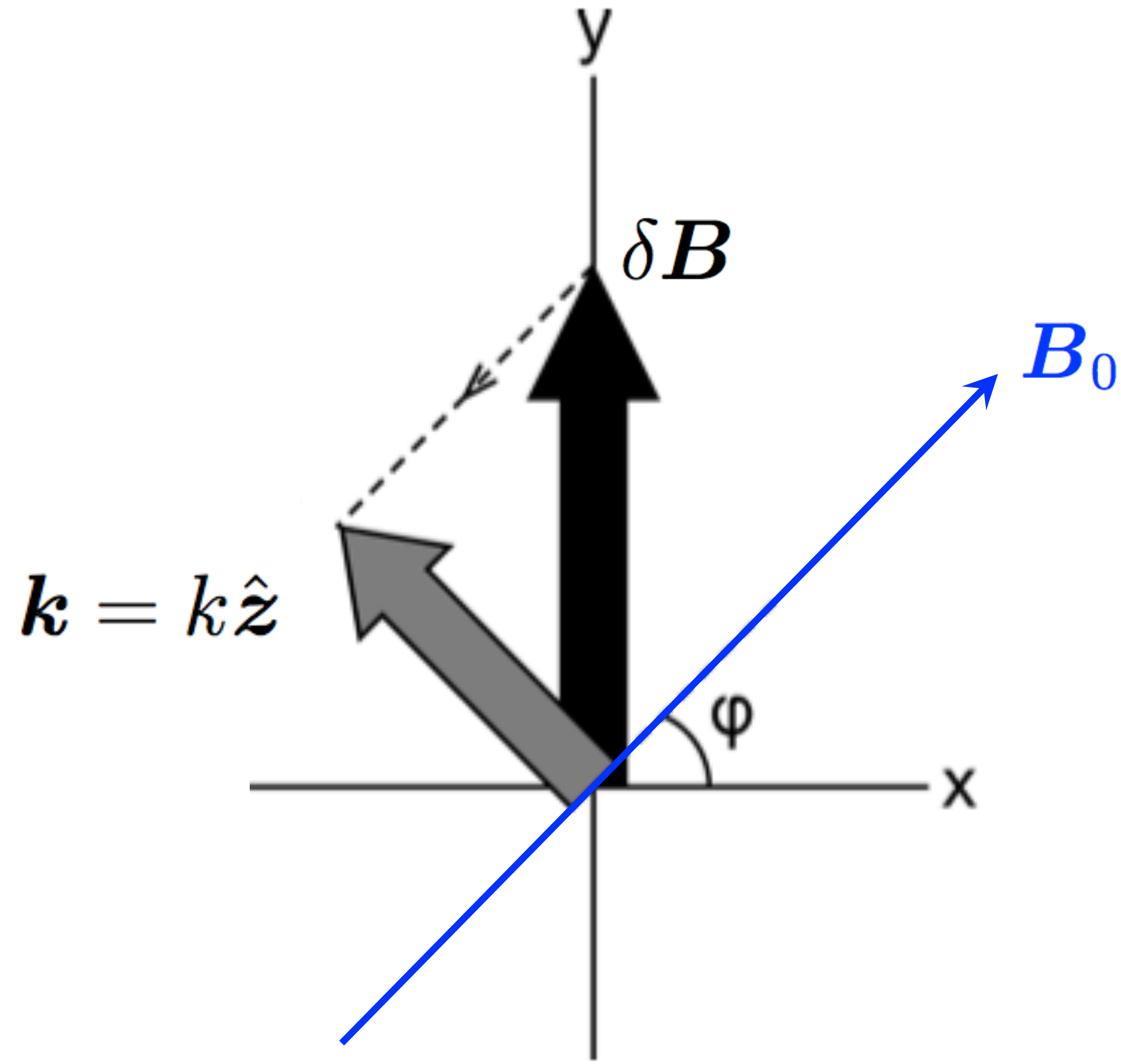


Ohmic dissipation



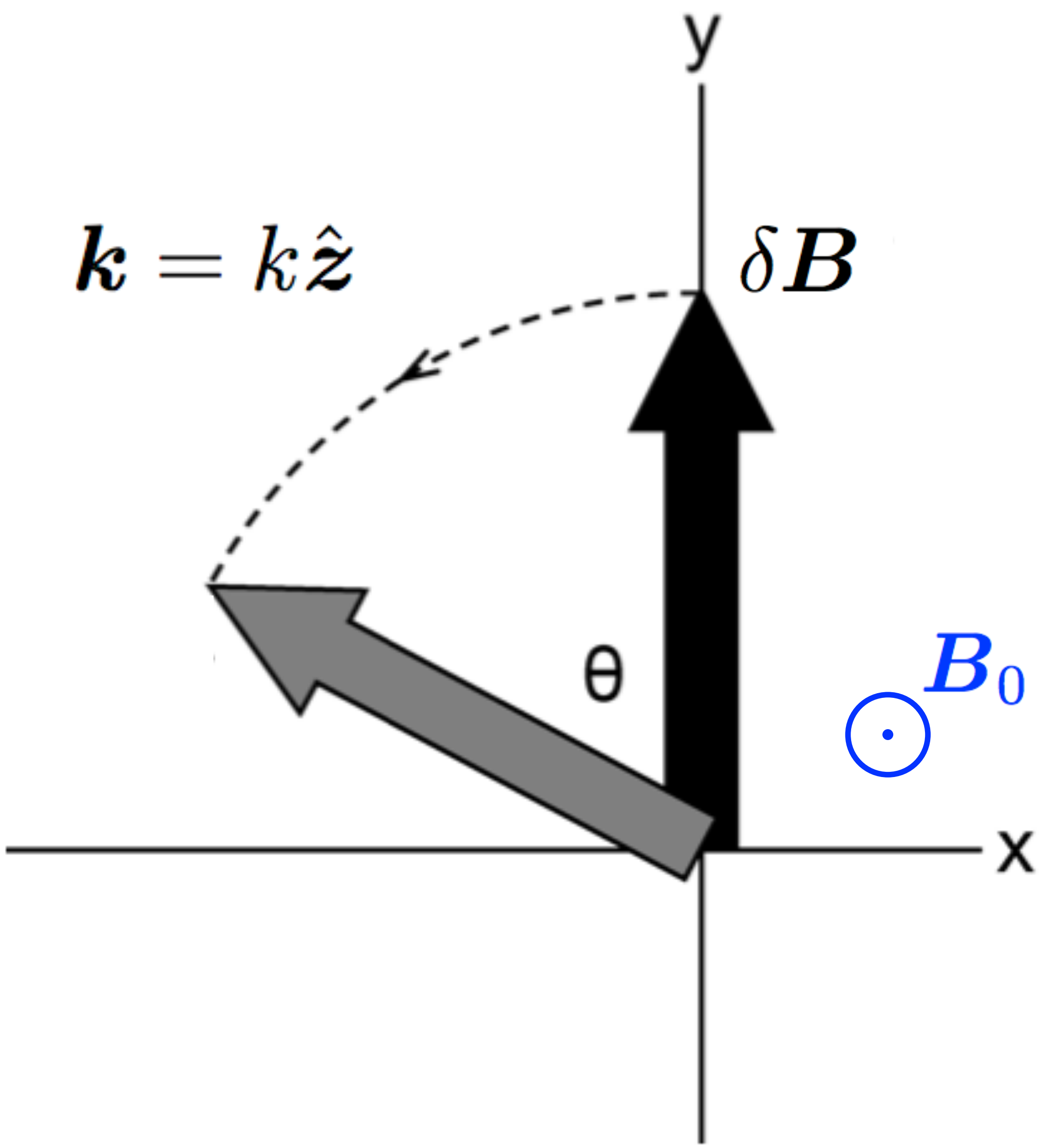
$$\partial_t \, \delta \boldsymbol{B} = \ldots - \boldsymbol{\nabla} \times (\eta_{\mathrm{OD}} \delta \boldsymbol{j})$$

ambipolar diffusion



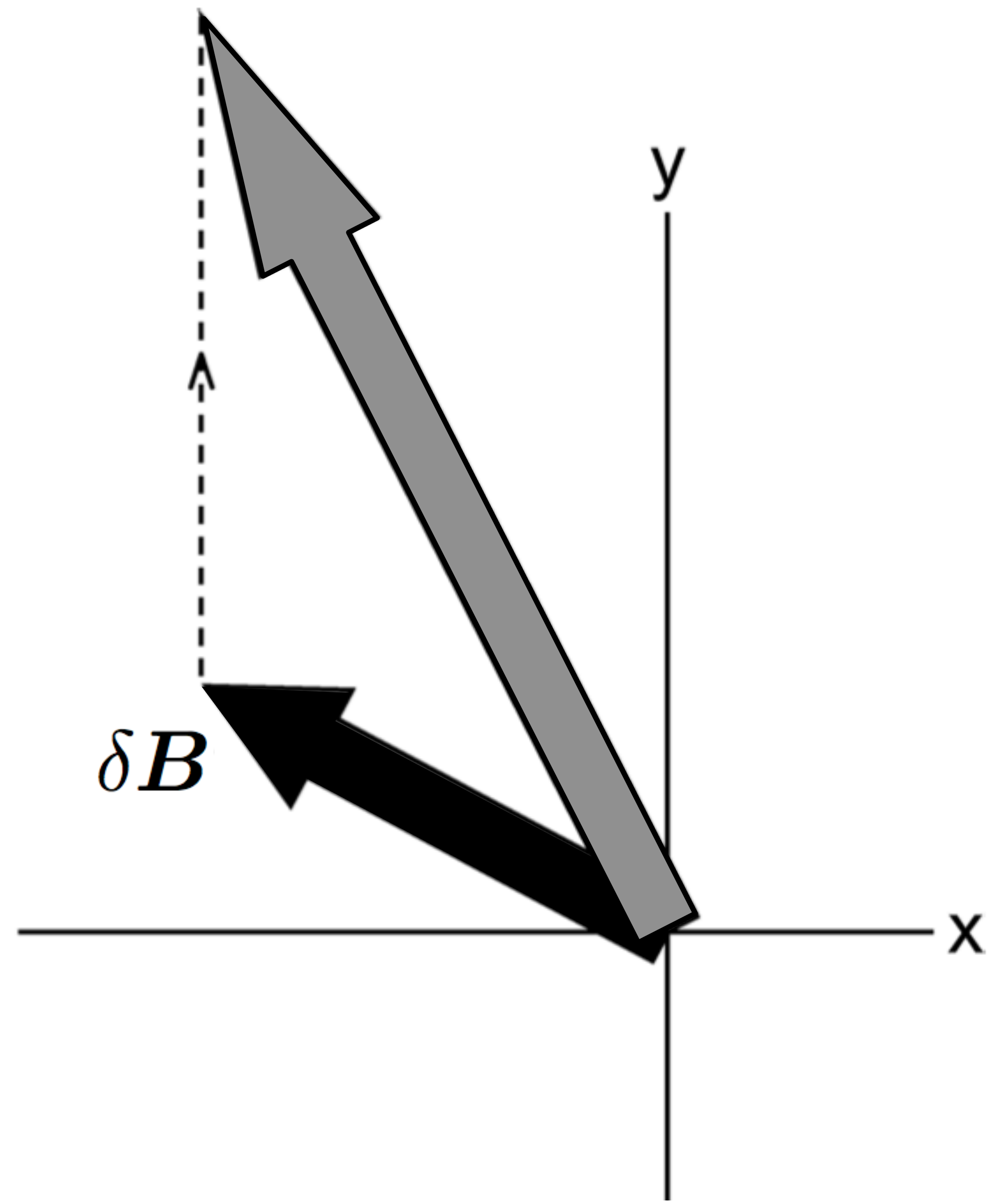
$$\partial_t \, \delta \boldsymbol{B} = \ldots - \boldsymbol{\nabla} \times (\eta_{\mathrm{AD}} \delta \boldsymbol{j}_\perp)$$

Hall effect



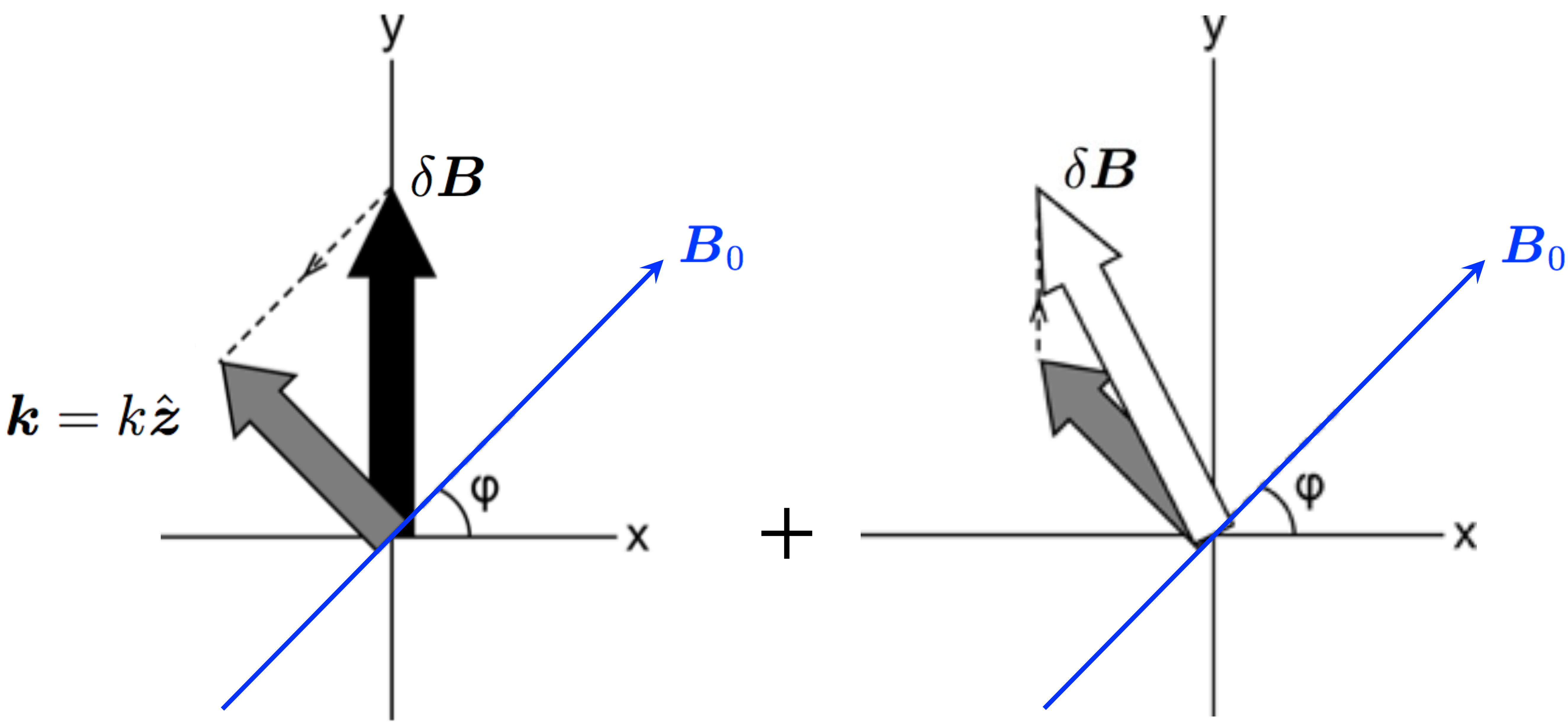
$$\partial_t\,\delta\boldsymbol{B} = \ldots - \boldsymbol{\nabla}\times(\eta_{\mathrm{H}}\delta\boldsymbol{j}\times\hat{\boldsymbol{b}})$$

Shear

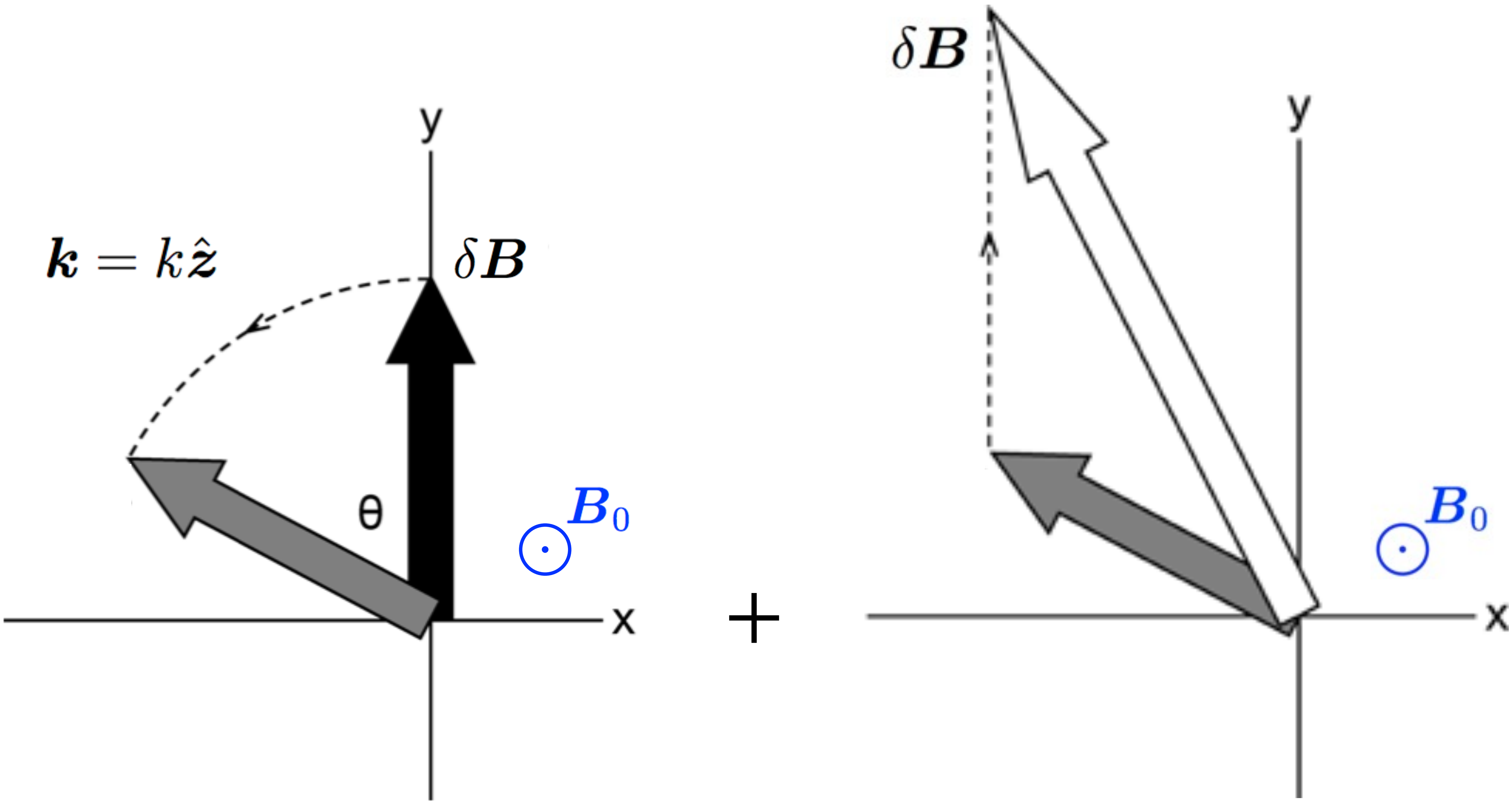


$$\partial_t\,\delta\boldsymbol{B} = \ldots - S\delta B_x\hat{\boldsymbol{y}}$$

Ambipolar-diffusion—shear instability (Kunz 2008)



Hall—shear instability (Kunz 2008)



Dispersion relation (Kunz 2008)

$$\gamma_+ \gamma_- \simeq -k^2 \boldsymbol{\eta} \boldsymbol{:} \boldsymbol{\nabla} \boldsymbol{v}$$

with

$$\gamma_\pm = \gamma + \frac{1}{2} k^2 \mathrm{tr}(\boldsymbol{\eta}) \pm \left[\frac{1}{4} k^4 \mathrm{tr}^2(\boldsymbol{\eta}) - k^4 \det(\boldsymbol{\eta}) \right]^{1/2}$$

AD: $\left(\gamma + k^2 \eta_{\mathrm{AD}}\right) \left(\gamma + k_{||}^2 \eta_{\mathrm{AD}}\right) \simeq \eta_{\mathrm{AD}} (\boldsymbol{k} \times \hat{\boldsymbol{b}})(\boldsymbol{k} \times \hat{\boldsymbol{b}}) \boldsymbol{:} \boldsymbol{\nabla} \boldsymbol{v}$

Hall: $\left(\gamma + \mathrm{i}\, k k_{||} \eta_{\mathrm{H}}\right) \left(\gamma - \mathrm{i}\, k k_{||} \eta_{\mathrm{H}}\right) \simeq -\eta_{\mathrm{H}} k_{||} \boldsymbol{k} \boldsymbol{\cdot} (\boldsymbol{\nabla} \times \boldsymbol{v})$

Anisotropic dissipation couples waves to free-energy gradients

$$\text{wave response} = \begin{cases} \hat{\boldsymbol{b}}\hat{\boldsymbol{b}} : \boldsymbol{\nabla v} \\ \hat{\boldsymbol{b}}\hat{\boldsymbol{b}} \cdot \boldsymbol{\nabla} T \\ (\hat{\boldsymbol{k}} \times \hat{\boldsymbol{b}})(\hat{\boldsymbol{k}} \times \hat{\boldsymbol{b}}) : \boldsymbol{\nabla v} \\ (\hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{b}})\ \hat{\boldsymbol{k}} \cdot (\boldsymbol{\nabla} \times \boldsymbol{v}) \end{cases}$$