

LETTER

New method for inductively forming an oblate field reversed configuration from a spheromak

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Abstract

A new method for inductively forming a field reversed configuration is demonstrated, based on the inductively driven transformation of a spheromak. The driven transition can be achieved in argon and krypton plasmas, in which MHD modes are suppressed; simulations indicate that stability through the transition is explained by magnetic diffusion. Spheromaks with lighter working gas, such as neon and helium, either display a tilt mode or an $n = 2$ kink instability, both resulting in discharge termination.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

The field reversed configuration (FRC) [1] is a unique toroidal plasma configuration with several important properties: the plasma is not linked by toroidal field coils, and has the highest attainable β (~ 1). These unique features have driven interest in the use of an FRC as the core of a fusion power system [1], as a fuelling source for other large magnetized plasmas [2] and as a test bed for studies of the basic physics of high- β plasmas [3, 4]. However, it has proven difficult to form and maintain these high- β plasmas. Traditional formation methods for axially elongated (prolate) FRCs have relied on fast formation using very high-voltage theta-pinch coils [1]; this technique has been augmented with the rotating-magnetic field (RMF) method [5]. For FRCs with a more spherical boundary (oblate FRCs), only formation through spheromak merging has found broad application [6–9].

In this letter, we report on the physics of a novel inductive method for forming a large-flux oblate FRC. When inductive current drive is applied to an argon or krypton spheromak in the magnetic reconnection experiment (MRX) [10], the transition to and subsequent sustainment of an FRC are observed to

occur; the central safety factor (q_0) goes from $q_0 > 0.5$ to $q_0 \approx 0$, and the paramagnetic spheromak transitions to a diamagnetic FRC. This transition does not occur in spheromaks formed in helium or neon; as q_0 drops below 1/2 in these cases, an $n = 2$ mode grows which terminates the plasma. MHD calculations indicate that the low Lundquist numbers in krypton and argon plasmas contribute to the stabilization of this mode. This result indicates a new technique to form an oblate FRC, provided that low- n kink instabilities can be avoided, and demonstrate that resistivity can provide this required stabilizing effect.

The transition between the spheromak and FRC configurations may be a surprising result, given the very different equilibria represented by these two states. The spheromak [11] is usually understood by the Taylor-equilibrium relationship [12]: $\vec{\nabla} \times \vec{B} = \lambda \vec{B}$, with λ a spatial constant. The Taylor condition implies that all currents are parallel to the magnetic field, the pressure gradient is small and the volume average β is low ($\beta = 2\mu_0 \int nT dV / \int B^2 dV$, with T and n the plasma temperature and density, B the magnetic field and $\int \dots dV$ the integral over the plasma volume). This spheromak equilibrium naturally leads to a configuration where the toroidal field has a maximum at the centre of the plasma and goes to zero at the edge, and where the poloidal field is maximized at the plasma edge. The classical FRC

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equilibrium, on the other hand, has only poloidal magnetic field and toroidal plasma current. This toroidal plasma current is purely diamagnetic ($\vec{J}_\perp = (\vec{B} \times \nabla p)/en$) and is everywhere perpendicular to the magnetic field. This equilibrium naturally leads to a configuration with β approaching unity.

These two very different equilibria have historically been distinguished by different formation techniques, sustainment methods and stability properties. However, results from the translation, confinement and sustainment (TCS) experiment, a prolate FRC facility, have demonstrated substantial toroidal field in prolate FRCs. These toroidal fields have been observed both after a theta-pinch formed FRC is translated into a confinement chamber [13] and during sustainment of an FRC by RMF [14]. Experiments in TS-3 [3] and TS-4 [4], which utilized the merging of two spheromaks to form an FRC, also created intermediate states between an FRC and a spheromak when the fields of the two spheromaks were not of nearly equal magnitude (but toroidal fields always of opposite sign), and then observed the subsequent relaxation to either a pure FRC or spheromak equilibrium. Based on TS-3 data [3], a threshold helicity was defined, above which the intermediate state relaxed to a spheromak, and below which relaxation to an FRC occurred. Further experiments in TS-4 reported that the threshold helicity was mass dependent [4], implying that two-fluid effects were important for relaxation to an FRC. However, because an intermediate state after merging was involved, the direct transition from a Taylor-equilibrium spheromak to an FRC was not observed. Indeed, those intermediate states near a spheromak equilibrium always relaxed to a spheromak, and those near an FRC relaxed to an FRC. We also note that the technique described here has similarities to that utilized for FRC formation in the coaxial slow source (CSS) experiment, though that scheme did not involve the transition from spheromak to FRC equilibria [15].

2. Experimental setup

The MRX [9] is a facility designed to study both the basic science of magnetic reconnection and the physics of compact toroid plasmas. This letter focuses on the case where two spheromaks with parallel toroidal fields are allowed to merge, forming a new spheromak (co-helicity merging). A 68-turn solenoid, composed of two separate 34-turn coils inserted from opposite directions, is installed along the geometric axis of the device. This coil sustained the plasma current by applying an inductive toroidal electric field to the plasma once merging is finished. There is a small separation between the solenoid coils at the midplane (~ 2 cm), through which a small amount of solenoid flux leaks. Additional compensating field coils are utilized in series with the solenoids to cancel the stray return flux of the solenoids.

The main diagnostics utilized in this study are two large arrays of magnetic pick-up coils. The large area array (see figure 1) is composed of a 7×6 grid of coil triplets (B_R , B_Z , and B_T), covering $0.22 \text{ m} < z < 0.22 \text{ m}$ and $0.11 \text{ m} < R < 0.52 \text{ m}$. This probe array, when combined with measurements of the solenoid flux from loops on the solenoid surface, is utilized to determine the equilibrium properties, such as the currents in the plasma, poloidal flux (ψ), toroidal flux (Φ), safety factor (q) and Taylor eigenvalue

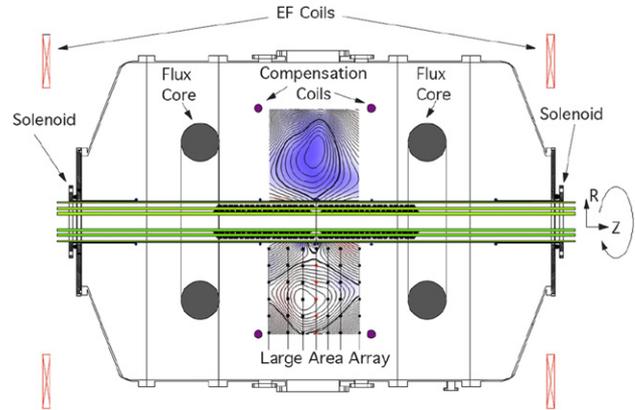


Figure 1. Schematic of the MRX device, additionally showing the poloidal flux (contours) and toroidal field (colours) of a typical spheromak (top) and FRC (bottom).

(λ). The second probe array consists of nine probes inserted at approximately equally spaced toroidal angles at the midplane. Each probe can measure all three components of the magnetic field at five locations radially separated by 8 cm. This array is utilized to study the growth of non-axisymmetries, and their underlying instabilities, with good time ($\sim 3 \mu\text{s}$) and spatial (up to $n = 4$ toroidal mode number resolution in B_R) resolution. The typical magnetic field strengths in these plasmas are 15–30 mT. Kinetic parameters are measured by a triple Langmuir probe. The electron temperature is in the range $T_e = 7\text{--}10 \text{ eV}$ throughout the discharge for both neon and argon cases.

3. Experimental results

A common fate for a deuterium or helium spheromak in MRX (once the initial merging is finished) is a terminal tilt instability [16]. This instability is caused by the tendency of the plasma to align its magnetic moment to the externally applied field. In principle, the rigid-body tilt mode can be suppressed by arranging for the external field to have curvature such that $n_{\text{decay}} = -(R/B_Z)dB_Z/dR > 1$ [17]. The present configuration of MRX typically had $n_{\text{decay}} \approx 0.5$ immediately after merging; the solenoid field compensation was selected so that $n_{\text{decay}} \approx 1$ during the solenoid ramp. Hence, the spheromak became more stable to the tilt as the sustained discharge progressed. Spheromak tilting has been observed in the past [17–19], and these tilted discharges will not be discussed further.

It was possible to reliably suppress the tilt instability under certain conditions. These cases occurred at higher fill pressure (10 mTorr in neon, marginal tilt suppression at 14 mTorr in helium), with strong externally applied field pushing the spheromak towards the solenoid. It is believed that some line tying to the liner and a lower growth time at high fill pressure and large plasma mass probably played a role in suppressing the $n = 1$ tilt. As an example of this behaviour, a scan of the solenoid capacitor-bank voltage was conducted (0, 3, 5, 7 and 9 kV), and the results are illustrated in figure 2. The solenoid flux is shown in figure 2(a), illustrating the magnitude of the inductive drive. The poloidal flux between the separatrix and the field null (the trapped flux, denoted ψ_t) is shown in figure 2(b). All cases where the solenoid was energized show

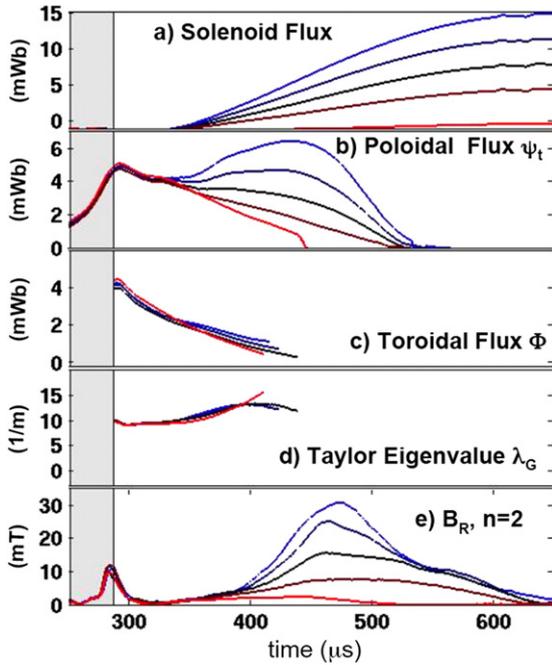


Figure 2. Evolution of neon spheromaks under inductive sustainment, for different voltages on the solenoid capacitor bank. The waveforms of λ_G and Φ are truncated when the growth of large non-axisymmetries causes data from the large area array to become unrepresentative. The grey area to the left represents the time before merging completion.

signs of sustainment, with the larger solenoid firing voltages leading to an increase in the poloidal flux. The toroidal flux, however, shows a monotonic decrease in time, as the poloidal currents that create it resistively decay. However, the plasmas always collapsed long before the end of the solenoid current ramp due to the growth of a terminal $n = 2$ mode, illustrated in figure 2(e). Additional analysis shows that this mode grows when the central safety factor (q_0) falls below $1/2$.

The global Taylor eigenvalue, λ_G , for these discharges is illustrated in figure 2(d), utilizing the definition $\lambda_G = \mu_0 \int \mathbf{J} \cdot \mathbf{B} dV / \int \mathbf{B} \cdot \mathbf{B} dV$. The expected value can be estimated [12] by treating the spheromak as if it were in a cylindrical flux conserver of length 0.35 m and radius 0.4 m; the resulting eigenvalue of 13 m^{-1} is close to the measured value of 10–12. Measurements of the λ profile show that it is slightly hollow after merging completion, and then becomes flat. When the solenoid is energized, the λ profile becomes peaked, and the discharge-ending $n = 2$ mode then develops. A similar $n = 2$ mode was observed in the S-1 spheromak following peaking of the λ -profile [20]. The mode caused a flux-conversion process and a consequent flattening of the λ -profile, i.e. relaxation towards a Taylor state. In that case, with nearby passive stabilizers, the plasma survived the instability. In the present case without stabilizers, the mode is always disruptive. A similar ‘relaxation oscillation’ was observed in CCTC-1 [21] and these modes have been predicted in MHD simulation [22].

This unstable behaviour in neon (and helium) is in stark contrast to the behaviour in argon and krypton. The poloidal flux (contours) and toroidal field (colours) for a typical argon discharge are illustrated in figure 3. The co-helicity merging is completed at $t = 280 \mu\text{s}$, and the system has settled to a large

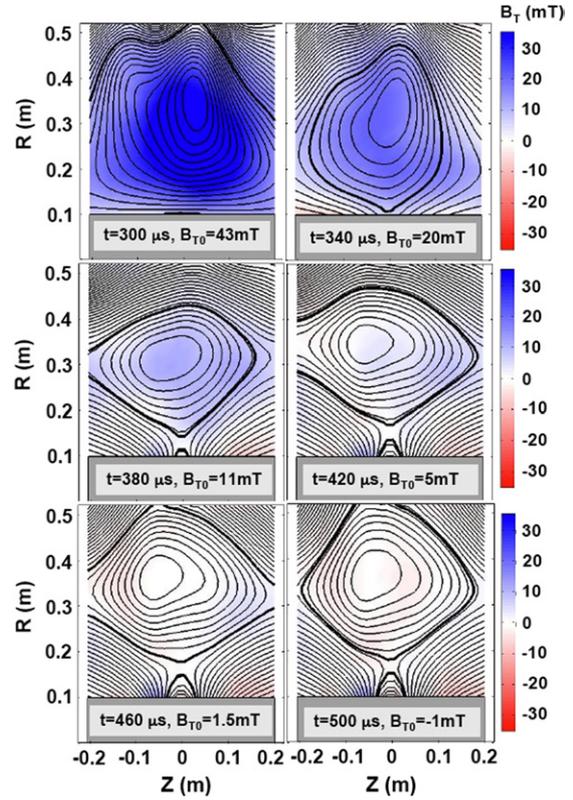


Figure 3. Two-dimensional measurements of the poloidal flux (contours) and toroidal field (colours, with central value listed in figure) during the transition of a spheromak to an FRC. The poloidal field at the outer separatrix decreases from 18 mT at $340 \mu\text{s}$ to 10 mT at $500 \mu\text{s}$.

spheromak configuration by $t = 300 \mu\text{s}$. This spheromak decays until the solenoid is energized at $t = 330 \mu\text{s}$, at which point the poloidal flux is increased and then sustained. The toroidal flux continues to resistively decay; however, no $n = 2$ kink is observed, and so the toroidal flux is allowed to decay to zero. The configuration thus ultimately makes a transition to an FRC equilibrium. Note that the final diamond shape is due to the combination of the leakage flux from the gap between the transformers and the pushing force from the solenoid compensating coils.

We measured the midplane T_e and n_e profiles in these argon plasmas by radially scanning the triple Langmuir probe during many repeatable discharges. There is a large increase in the plasma pressure (by a factor of ~ 2) during the solenoid ramp, forming the peaked profile consistent with the FRC diamagnetic equilibrium. This rise is mainly due to an increase in the density, from $n_{e0} = 1 \times 10^{20} \text{ m}^{-3}$ to $n_{e0} = 1.7 \times 10^{20} \text{ m}^{-3}$, as additional magnetic flux and particles are brought in by the induction driven $E_\phi \times B_p$ inward drift (n_{e0} is the density at the poloidal field null). The plasma temperature shows only a slight ($\sim 15\%$) increase at the start of the sustained phase, but is maintained at a constant level throughout the solenoid pulse. This pressure rise, coupled to the reduction in magnetic field energy as the toroidal fields decay and the poloidal fields resettle to a somewhat lower level, results in the conversion of the $\beta \sim 0.1$ – 0.2 spheromak to the $\beta \sim 0.8$ – 1 diamagnetic FRC equilibrium (here β is estimated from the magnetic measurements using

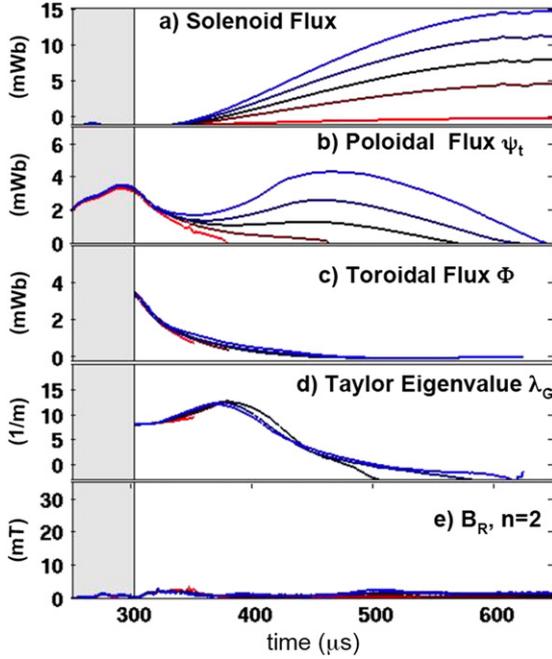


Figure 4. Time evolution of argon spheromaks under sustainment, for different voltages on the solenoid capacitor bank, demonstrating the transition to an FRC.

force balance). This FRC equilibrium is then sustained as a balance between the $E \times B$ drift of incoming particles and magnetic flux ($\propto E_\phi/B_p$) and the outward diffusion of those quantities ($\propto \eta \partial P / \partial \psi$). Increasing the $E_\phi \times B_p$ inward pinch with a faster solenoid ramp will lead to a larger pressure gradient to balance the inward pinch, and thus a larger toroidal current ($J_\phi \propto R \partial P / \partial \psi$) and trapped flux. The details of this inductive sustainment of an FRC are described in much greater detail in [10].

Many single-parameter scans have been made in order to understand this behavior; the results of a scan over the solenoid firing voltage is illustrated in figure 4, where the voltages and parameters plotted are the same as those in figure 2. The solenoid flux is shown in figure 4(a), the trapped poloidal (ψ_t) and toroidal (Φ) flux in figures 4(b) and (c), and the Taylor eigenvalue (λ_G) in 4(d). For the case with no induction, the poloidal and toroidal fluxes decay at the same rate, and λ_G is approximately constant through the lifetime of the plasma with a value near the Taylor-state expectation; this closely mimics the observation in neon plasmas without induction. When induction is applied, the poloidal flux is either maintained or increased, depending on the magnitude of the solenoid voltage. The toroidal flux decay, however, is essentially unchanged by the varying solenoid voltage; both λ_G and Φ decay to zero even as the poloidal flux increases, and an FRC state is formed. Besides the drop in Φ and λ_G , the most striking feature of these discharges is the lack of a B_R $n = 2$ perturbation, even as q_0 drops below 0.5. The evolution of these plasmas thus appears to be purely two dimensional. The evolution of the λ profile in this argon case is similar to that in neon, except that the peaking during the early part of the solenoid ramp is followed by a drop of the profile to near zero everywhere.

Other single-parameter scans have been performed, all of which have illustrated the robust nature of this transition.

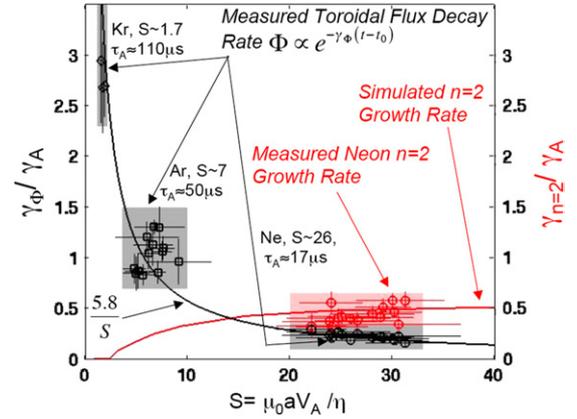


Figure 5. Comparison between the computed $n = 2$ growth rate, the measured neon $n = 2$ growth rate, the toroidal flux decay rate and the function of $5.8/S$. The approximate Alfvén times are also indicated.

Scans over the solenoid firing time show that the transition always occurs, regardless of the spheromak poloidal flux level when the induction begins. Scans over fill pressure have been done; the transition is observed to occur somewhat more quickly at high fill pressure. The direction of the toroidal fields of the spheromak has been reversed; no significant change was observed in the dynamics of the transition. Spheromaks formed in krypton show nearly identical transition properties.

4. Discussion and conclusions

The argon and krypton spheromaks maintain stability to $n = 2$ modes (and all other n as well) as the central q drops to zero. A likely reason for the stability in these plasmas is the small Lundquist number $S = \mu_0 a V_A / \eta$: the perturbed currents that drive the instability are resistively dissipated faster than the instability can grow (note that the equilibrium current is maintained by the steady poloidal flux injection from the solenoid). In order to test this hypothesis, we have used the resistive MHD version of the HYM code [23] to calculate the growth rate of spheromak $n = 2$ modes as a function of Lundquist number. The inductive sustainment was modelled by eliminating the $n = 0$ component of the resistivity, while preserving resistivity for perturbations. The growth rates in the calculation are normalized to the ideal-mode growth rate of $\gamma_A = V_A/a$, with $V_A = (B_{Z,sep}^2 / (\mu_0 \rho))^{1/2}$, a the plasma minor radius and ρ the plasma mass density.

The results from the calculation are shown in figure 5, where the $n = 2$ growth rate is plotted as a function of S . For large S , the $n = 2$ kink has a normalized growth rate of ~ 0.5 ; the growth rate quickly decreases with increasing resistivity, and stability is predicted for $S < 2.5$. The measured $n = 2$ growth rate for the neon discharges is shown as a set of open circles, and good agreement with the simulation is found. Also shown in squares is the toroidal flux decay rate (γ_Φ/γ_A), derived by fitting the formula in the figure to the measured $\Phi(t)$. The growth rate of the $n = 2$ mode in neon exceeds γ_Φ/γ_A , allowing the instability to terminate the discharge before the transition occurs. For argon, γ_Φ/γ_A is approximately 2–4 times the predicted $n = 2$ growth rate; the plasma achieves an FRC equilibrium on a time scale faster than

the $n = 2$ kink is predicted to grow. The krypton plasmas are predicted to be absolutely stable to the $n = 2$ mode. The measured γ_Φ/γ_A shows a $5.8/S$ scaling, as expected for magnetic diffusion [24]. Note, however, that the role played by resistivity in maintaining kink stability is probably not fundamental; any effect outside of ideal MHD (resistivity, shear-flows, finite-Larmor radius (FLR) stabilization, etc) which maintains stability to these kink instabilities for times longer than the toroidal flux decay time would probably allow the transition. Also note that the final FRC plasma is unstable in ideal MHD, due to the everywhere bad curvature of the magnetic field lines [25]; effects discussed in [10], including the shaping of the equilibrium field, resistivity and FLR effects, likely contribute to the observed stability.

These results have important implications for the understanding of FRC formation and equilibrium. Spheromak formation via co-helicity merging was utilized for convenience in MRX; however, it should be possible to use a single flux-core (or indeed another formation scheme) to produce a single spheromak, which could then be ‘converted’ to an FRC utilizing solenoid induction. As indicated by the poloidal flux rise in figure 4, the resulting FRC could have a large amount of poloidal flux, while the ohmic dissipation of the poloidal currents could provide some heating. The FRC could then be translated away from this formation region and into a simply connected ‘confinement’ region. This scheme allows for a ‘black-box’ plasma source capable of forming either a low- β spheromak or a high- β FRC, depending on the utilization of the solenoid, with the exact same hardware. The results also demonstrate a situation where the FRC is clearly a preferred state: when stability to the low- n MHD kinks is maintained for a period longer than the resistive decay time, yet the equilibrium poloidal flux is externally maintained.

In summary, inductive current drive has been utilized to convert a spheromak into an FRC. The transition occurs robustly in argon and krypton, where resistivity effects are important in stabilizing the low- n kinks. This scheme indicates

a new and possibly simpler mechanism to form an oblate FRC, provided that stability to the $n = 1$ and 2 spheromak instabilities is achieved during the transition.

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