An electromagnetic drift instability in the magnetic reconnection experiment and its importance for magnetic reconnection[a)]

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The role which resistivity plays in breaking magnetic field lines, heating the plasma, and plasma-field slippage during magnetic reconnection is discussed. Magnetic fluctuations are observed in the MRX (magnetic reconnection experiment) [M. Yamada, H. Ji, S. Hsu, T. Carter, R. Kulsrud, N. Bertz, F. Jobes, Y. Ono, and F. Perkins, Phys. Plasmas **4**, 1936 (1997)] that are believed to provide resistive friction or wave resistivity. A localized linear theory has been proposed for their origin as an obliquely propagating lower hybrid drift instability. In this paper, the linear theory of the instability is summarized, and the resulting heating and slippage are calculated from quasilinear theory. Making use of measured amplitudes of the magnetic fluctuations in the MRX, the amount of these effects is estimated. Within the experimental uncertainties they are shown to be quite important for the magnetic reconnection process.
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I. INTRODUCTION

For a long time, it has been known that magnetic reconnection in nature proceeds much faster than given by the Sweet-Parker model.[1,2] It is gratifying that this also occurs in the magnetic reconnection experiment (MRX) at Princeton, which is a dedicated experiment to study the fundamental physics of magnetic reconnection.[3] Recently, electromagnetic fluctuations have been identified[4] in the MRX, and a theoretical explanation has been found for them.[5] These fluctuations have the potential to play a significant role in reconnection physics as they may enhance the normal plasma resistivity and speed up the reconnection process. Thus, the MRX provides us with the opportunity to directly unravel the cause of this speed up.

The theory of reconnection has been based on the Sweet-Parker model that intrinsically involves the resistivity η. Thus, it has been generally assumed that resistivity is necessary to break and reconnect magnetic lines of force. The reason for this belief is most easily understood from an examination of the region around the X line, where the lines are visibly breaking as in Fig. 1.

Here lines are passing from the unreconnected region A to the reconnected region B. The reduction of lines in region A generates an electric field along the X line, whose magnitude is proportional to the rate of transfer of lines from A to B, the reconnection rate. This electric field tends to accelerate any electron along the X line. Since $\mathbf{B}=0$ on the X line, the only magnetohydrodynamics (MHD) force resisting this acceleration is the resistivity η. If η is small, it would seem that the reconnection rate must therefore also be small, otherwise the current along X would be large. However, electrons do not simply sit at rest along X, but move rapidly along the field lines spending only a short time near the X line. After they reach a region of appreciable magnetic field strength, say after traveling a distance d, the electric force is resisted by the magnetic force and the acceleration stops. If the distance d is short compared to the mean-free path λ then the time of acceleration is short compared to the electron-ion collision time, and the effective resistance is increased by λ/d. That is, from the point of view of line breaking, the resistivity appears to be enhanced by this factor.

This is borne out by numerical simulations of collisionless reconnection. The term in the generalized Ohm's law

$$\mathbf{E}+\frac{\mathbf{v}_e\times\mathbf{B}}{c}=-\frac{\nabla\cdot\mathbf{P}_e}{ne}+\eta\mathbf{j} \tag{1}$$

that represents the above effect is the pressure tensor $\nabla\cdot\mathbf{P}_e$ terms.[6] [The ratio of $\nabla\cdot\mathbf{P}_e\approx mnv_Dv_{\rm th}/d$ to $\eta\mathbf{j}ne\approx\nu mnv_D$ is $v_{\rm th}/\nu d\approx\lambda/d$.]

Clearly the frictional force ($\nabla\cdot\mathbf{P}_e$ or $\eta\mathbf{j}$) at the X line, where the two components of the poloidal field B_p (B_x and B_y) vanish, is of fundamental importance, since it determines the toroidal component of $\mathbf{E}$ (in terms of v_D or j). However, is the resistivity really important elsewhere, where $B_p\neq 0$? At such a place, ignoring $\nabla\cdot\mathbf{P}_e$ Ohm's law can be written as

$$\mathbf{E}+\frac{(\mathbf{v}_e+\mathbf{v}_\eta)\times\mathbf{B}}{c}=0, \tag{2}$$

where $\eta\mathbf{j}=-(\mathbf{v}_\eta\times\mathbf{B})$, so that the line velocity cE/B differs from the plasma velocity by $\mathbf{v}_\eta$. The direction of $\mathbf{v}_\eta$ is such that the magnetic lines move through the reconnection layer faster than the plasma. At most places the slippage is of the order of or smaller than $\mathbf{v}_e$, so that the velocity pattern is not appreciably changed.

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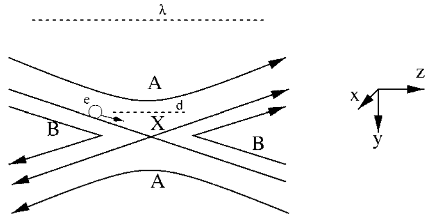

FIG. 1. The small region around the X line.

Thus in the case when $\nabla \cdot \mathbf{P}_e$ is greater than $\eta \mathbf{j}$ at the X line, one might expect the resistivity to be of little consequence to the reconnection process.

However, resistivity does serve two other functions in the reconnection process. (a) The first function is in heating the plasma. If there is no guide field parallel to the X line, the plasma must have an increased pressure in the reconnection layer to balance the decreased (poloidal) magnetic pressure. This increase can only arise from heating of some sort. (b) Now without collisions the main heating must come from adiabatic compression associated with a rise in density in the layer. But without resistivity there can be no compression transverse to the magnetic field, since (without parallel compression) B/n is constant for a constant length fluid element as it passes into the layer and B clearly decreases. This must be at least true for any electron fluid element and therefore, by charge neutrality, for any ion fluid element also. Thus, the second function of resistivity is the v_η slippage between the field and the plasma which breaks the B/n constraint and allows B to decrease without forcing n to decrease.

In any normal reconnection occurring in nature there is plasma expansion longitudinally along the field to get the plasma out of the way of more incoming unreconnected plasma. This would increase B/n rather than decrease it. (However, this need not be the case for numerical simulations of collisionless reconnection with periodic boundaries. One expects longitudinal compression as plasma accumulates in the reconnection layer. This could be responsible for the heating in numerical simulations.[7,8])

These two functions indicate the important role that resistivity plays in reconnection processes, and it is difficult to see how reconnection can properly function without it.

If the only nonideal MHD reconnection process were resistivity then reconnection of field lines should proceed at the Sweet-Parker reconnection velocity

$$v_R = v_A \sqrt{\frac{\eta}{L v_A}} = \frac{v_A}{\sqrt{S}}, \tag{3}$$

where v_A is the Alfvén speed, $S = L v_A / \eta$ is the Lundquist number, v_R is the upstream velocity of lines coming into the reconnection layer, and L is the length of the layer. Since in solar and space plasmas L is very large, v_R is generally small and too slow to explain the observations.

A measurement of the effective resistivity η^* has been carried out in the MRX by taking the ratio of E to j at the X line. The effective resistivity is found to increase relative to the Spitzer resistivity as the background density is lowered: At the lowest densities it reaches a value ten times the perpendicular Spitzer resistivity. From Eq. (1), we see that this ratio gives the correct value of η at a point where $B=0$ only if the $\nabla \cdot \mathbf{P}_e$ term is ignored. Earlier this was assumed and the effective resistivity was substituted in the Sweet-Parker equation (3). The measured reconnection rate is then found to be in reasonable agreement with Eq. (3) with an effective Lundquist number based on η^*, and provided, further, that v_A is replaced by the measured downstream velocity v_z which is considerably smaller.[9]

Moreover, it is also found that when the measured value of j is combined with the measured value of n_e, the relative drift velocity of the electrons with respect to the ions is in the range of a two or three the ion sound velocity. This suggests the presence of a plasma instability, which could excite waves and produce this enhancement in the resistivity. A primary purpose of this paper is to quantitatively evaluate the amount of enhanced resistivity generated by electromagnetic fluctuations in the MRX (Ref. 4) and compare it with the collisional resistivity.

II. FLUCTUATIONS IN THE RECONNECTION LAYER

A search was made for unstable fluctuations and indeed electrostatic fluctuations were first found by Carter *et al.*[10] However, it turned out that these fluctuations are not present throughout the layer but exist only on its edge. Further, they do not correlate well in time with the reconnection process. The role of these electrostatic fluctuations in the MRX is still not clear, although it was conjectured that they can make the current layer thinner and trigger fast reconnection.[11,12] Subsequently, Ji *et al.* uncovered[4] electromagnetic fluctuations of appreciable amplitude that are indeed present throughout the reconnection layer and do appear to correlate well in time with the reconnection process. On the basis of these observations, a local linearized electromagnetic instability theory has been developed[5] which shows how these fluctuations could arise.

It is surprising that this instability has not been commented on more frequently in the many papers devoted to the lower hybrid drift instability[13] as discussed in our previous paper.[5] We find that such an instability has actually been considered before in the papers on the modified two-stream instability,[14] but its importance has been disregarded because the current in the instability is not diamagnetic. We conjecture that the main reason for this lack of attention to it is that, in the cases treated, the perpendicularly propagating mode is the dominant one and the obliquely propagating modes grow more slowly. However, in the central region of the MRX current layer, the perpendicular mode is stabilized by the large magnetic field gradients, and only the oblique modes are unstable.

We will now describe this theory, and its quasilinear extension, that derives the force on the electrons and thus the enhancement in resistivity. We will also show how one can estimate the amount of heating due to these waves. The amplitudes of the magnetic fluctuations in these waves have been carefully measured. It should be emphasized that the waves are present throughout the reconnection layer. Assuming we have the correct theory we can determine how much

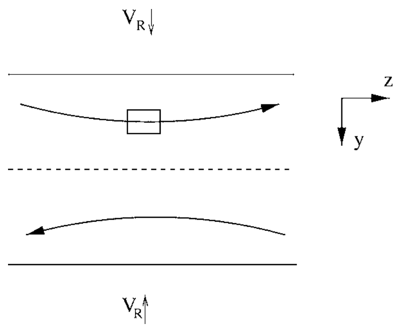

FIG. 2. The small region in which the local wave is described.

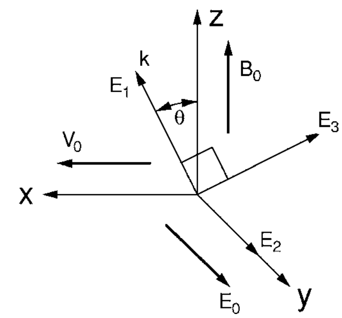

FIG. 3. The orientation of the electrostatic component E_1 and the electromagnetic components E_2 and E_3.

resistivity they produce independent of any reconnection theory. Alternatively, we can also find out if, within the uncertainties of the measurements and the crudeness of the theory, the waves could possibly be the main agent of the reconnection process.

Because of the considerable uncertainty in the measurements and also in the reconnection theory, it seems advisable to first work with the simplest and most flexible theory of this instability rather than to try to reach a high degree of accuracy with a complicated numerical calculation. This simplification is useful because of the large number of parameters involved in the instability. In fact, the simple theory does lead to a reasonable assumption that the instability is important, and it is now appropriate to carry out a more detailed numerical solution of it, at least for a limited choice of values of the parameters.

III. LOCAL LINEAR THEORY

In the reconnection layer shown in Fig. 2, let us concentrate on a small volume such as that shown by the little rectangle. As just mentioned, the thrust of the discussion will be to keep the analysis as simple as possible. This is the main motivation for a localized theory. However, it turns out that the growth rate of the waves is so large that we do not expect the waves to propagate very far before they reach a nonlinear state. [For an initial-value problem, it takes a finite time for the perturbation to settle down to the fastest-growing (nonlocal) eigenmode. If the local growth rate is so fast that the perturbation saturates nonlinearly in this time then one does not expect the perturbation to become an eigenmode in the linear regime.]

A more complete local theory of this instability has been presented in another paper,[5] and here we only give a summary of it sufficient for its quasilinear extension.

Therefore, we assume that the equilibrium plasma quantities in this region are uniform except that there is a radial electric field E_0 and a constant relative electron-ion drift U associated with the equilibrium current. We discuss the instability in the ion rest frame in which the ions are at rest. Thus, in zero order

$$\nabla p_i^0 = n_0 e \mathbf{E}_0,$$

$$\nabla p_e^0 = - n_0 e \mathbf{E}_0 - n_0 e \left(\frac{\mathbf{V}_0 \times \mathbf{B}_0}{c} \right). \tag{4}$$

We assume charge neutrality, and also that T_e and T_i are uniform but can be unequal.

Thus, in this frame the ion pressure is confined only by an electric field and the electron pressure is confined against this electric field by its electric current. We take a local Cartesian coordinate system with z along B_0, y in the direction of increasing plasma pressure (towards the central reconnection plane), and x in the negative current direction and in the direction of electron drift. Thus, $\mathbf{E}_0 = E_0 \hat{\mathbf{y}}$, $\mathbf{V}_0 = V_0 \hat{\mathbf{x}}$. From Eq. (4), we can show that

$$E_0 = \frac{T_i}{T_i + T_e} \frac{V_0 B_0}{c}. \tag{5}$$

Now for the local linearized perturbations, we assume that the electric field has the perturbation,

$$\mathbf{E} = \mathrm{Re}[\hat{\mathbf{E}} e^{i(k_x x + k_z z - i\omega t)}], \tag{6}$$

where we take $k_y = 0$. Since the mode is electromagnetic, it is convenient to use an electrostatic component E_1 and two electromagnetic components E_2 and E_3 as the fundamental variables for the perturbation. (E_1 is along $\mathbf{k}$, E_2 is along the y axis, and E_3 is in the $x-z$ plane and perpendicular to $\mathbf{k}$, as in Fig. 3.)

To keep the discussion as simple as possible we make the assumptions that for the perturbation the ions are unmagnetized and cold, and that the electrons can be treated by the drift kinetic theory. Namely, we assume that to lowest order the perturbed motion of the electrons is confined to perturbed magnetic field lines, and that their current can be obtained from the first moment of the Vlasov equations, i.e., the equation of motion but neglecting the electron inertia term. Instead of solving the reduced Vlasov equation to obtain the pressure tensor, we simply assume that their pressure is isothermal and isotropic. (The isothermal assumption is justified because the parallel heat flow is fast. The isotropic assumption is reasonable because for most cases the collision rate is comparable to the wave frequency.)

The electron equation of motion is

$$\mathbf{j}_{1e} \times \mathbf{B}_0 + \mathbf{j}_{0e} \times \mathbf{B}_1 - n_1 e \mathbf{E}_0 - n_0 e \mathbf{E}_1 - \nabla (n_1 T_e) = 0. \quad (7)$$

Here $\mathbf{j}_{0e} = -n_0 e \mathbf{V}_0$, We find $\mathbf{j}_{1e}$ from Maxwell's equations

$$-\mathbf{k} \times (\mathbf{k} \times \mathbf{E}) = \frac{4\pi i \omega}{c} (\mathbf{j}_{1e} + \mathbf{j}_{1i}), \quad (8)$$

where we use the cold ion equations to obtain $\mathbf{j}_{1i} = i n_0 e^2 \mathbf{E}/M\omega$. The cold ion assumption is rather inaccurate. It is shown[5] that including warm ions reduces the growth rate by as much as a factor of 2, but preserves the instability. However, the warm ion mode equations are complicated. Since there are both experimental uncertainties and other theoretical uncertainties, we consider it appropriate to treat the instability by the consistent set of mode equations based on the cold ion approximation. The perturbed magnetic field is $\mathbf{B}_1 = \mathbf{k}c \times \mathbf{E}/\omega^2$. Only the electromagnetic components of $\mathbf{E}$, E_2, and E_3, enter into this equation. From charge neutrality and the cold ion equations, we observe that the perturbed electron density is equal to the perturbed ion density, which is given by $n_1 = i n_0 e \mathbf{k} \cdot \mathbf{E}/M\omega$, which in turn is obtained from the continuity equation for the ion density with a term proportional to background density gradient missing. In the companion paper,[5] we showed that this term only produces very small changes in the growth rate. Thus, only the electrostatic component E_1 enters into the expression for the perturbed density.

The basic equations are obtained by taking x, y, and z components of the electron force equation (7). After some manipulations, we obtain

$$\left(\Omega \sin\theta - KV\frac{\beta_i}{\beta_e + \beta_i}\right)\frac{E_1}{\Omega} + i(\Omega - KV\sin\theta)E_2 - (K^2+1)\cos\theta E_3 = 0, \quad (9)$$

$$-i\sin\theta\left(\Omega^2 - \frac{\beta_e}{2}K^2\right)\frac{E_1}{\Omega} + (K^2+1)E_2 + i\Omega\cos\theta E_3 = 0, \quad (10)$$

$$\cos\theta\left(\Omega^2 - \frac{\beta_e}{2}K^2\right)\frac{E_1}{\Omega} + (\Omega\sin\theta - KV)E_3 = 0, \quad (11)$$

where the dimensionless quantities K, V, and Ω are defined by

$$k = K\omega_{pi}/c, \quad \omega = \Omega\omega_{ci}, \quad V_0 = VV_A, \quad (12)$$

where c/ω_{pi} is the local ion skin depth, ω_{ci} is the local ion cyclotron frequency, and V_A is the local Alfvén speed. Also $\beta_e = 8\pi n_0 T_e/B_0^2$ and $\beta_i = 8\pi n_0 T_i/B_0^2$ are the electron and ion β's.

The determinant of these equations is a quartic equation in Ω,

$$\Omega^4 - 2KV\sin\theta\Omega^3 - \left[(K^2+1)(K^2\cos^2\theta+1) - K^2V^2\sin^2\theta + \frac{\beta_e}{2}K^2\right]\Omega^2 + KV\sin\theta\left[\beta_e K^2 + (K^2+1)\frac{\beta_e + 2\beta_i}{\beta_e+\beta_i}\right]\Omega + K^2\left[\frac{\beta_e}{2}[(K^2+1)^2\cos^2\theta - K^2V^2\sin^2\theta] - (K^2+1)V^2\frac{\beta_i}{\beta_e+\beta_i}\right] = 0. \quad (13)$$

If the equilibrium is homogeneous, $V=0$, this quartic equation factors. If $\theta=0$, it factors as

$$[\Omega^2 - (K^2+1)^2]\left(\Omega^2 - \frac{K^2\beta_e}{2}\right) = 0, \quad (14)$$

whose roots are the two whistlers and the two sound waves[15] (as can be seen by returning to dimensional variables). Thus, the four roots of the quartic are modifications of the familiar whistlers and the ion acoustic modes. As we shall see, for appropriate ranges of K and V, the modification can lead to one unstable, one damped, and two propagating modes. We believe that it is this unstable mode that gives rise to the observed fluctuations in the MRX.

One can get some insight into the coupling of the modes that leads to instability in the exact solutions of the quartic, by passing to the limit of large K and V. Then keeping the dominant terms in each of the coefficients of the quartic, one can find asymptotic solutions to the quartic in the usual way by balancing terms. Two solutions for Ω result from balancing the quartic, cubic, and quadratic terms. Solution of the resulting quadratic, after canceling Ω^2, are $\Omega = K_x V \pm KK_z$ which in dimensional variables is

$$\omega = kV_0\sin\theta \pm \frac{k^2V_A^2}{\omega_{ci}}\cos\theta. \quad (15)$$

These are the two whistler modes Doppler shifted in the ion frame by the electron drift motion V_0. The other two solutions come from balancing the quadratic, and constant terms of the quartic and are

$$\Omega^2 = K^2\frac{(\beta_e/2)K^2\cos^2\theta - [(\beta_e/2)\sin^2\theta + \beta_i/(\beta_e+\beta_i)]V^2}{K^2\cos^2\theta - V^2\sin^2\theta}. \quad (16)$$

For $V=0$ this reduces to the sound mode. But for K in the range $[V\tan\theta, V\sqrt{\tan^2\theta + 2\beta_i\sec^2\theta/[\beta_e(\beta_e+\beta_i)]}]$, the right-hand side is negative and there are two complex conjugate roots, one of which is unstable.

The exact behavior is given in Fig. 4 where all four roots are plotted for $V=6$, $\theta=60°$, and $\beta_e=\beta_i=1$. We see that for some values of K there is one unstable root and for other values the four roots are all real and the four modes are all stable.

Note that, for fixed V, the lower limit on K for instability corresponds to the K at which the backward propagating Doppler shifted whistler just vanishes. Actually in the more general nonasymptotic state this occurs when this whistler has the same frequency as the sound mode. It seems that

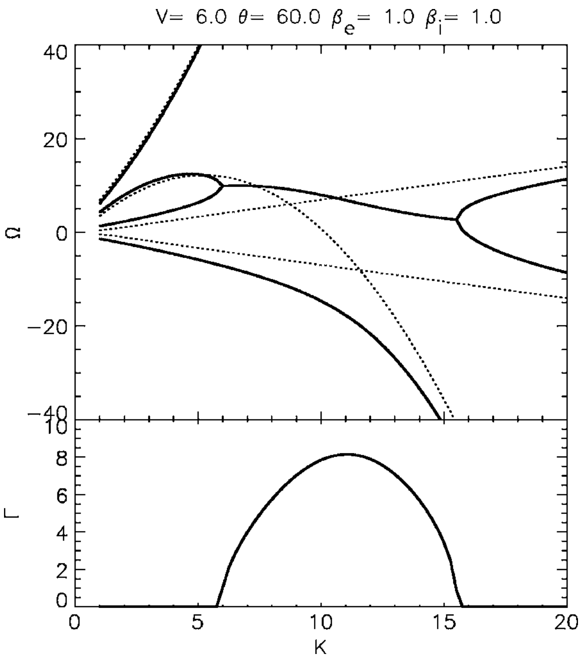

FIG. 4. The four roots of the quartic as a function of the normalized wave number $K=k\delta_i$.

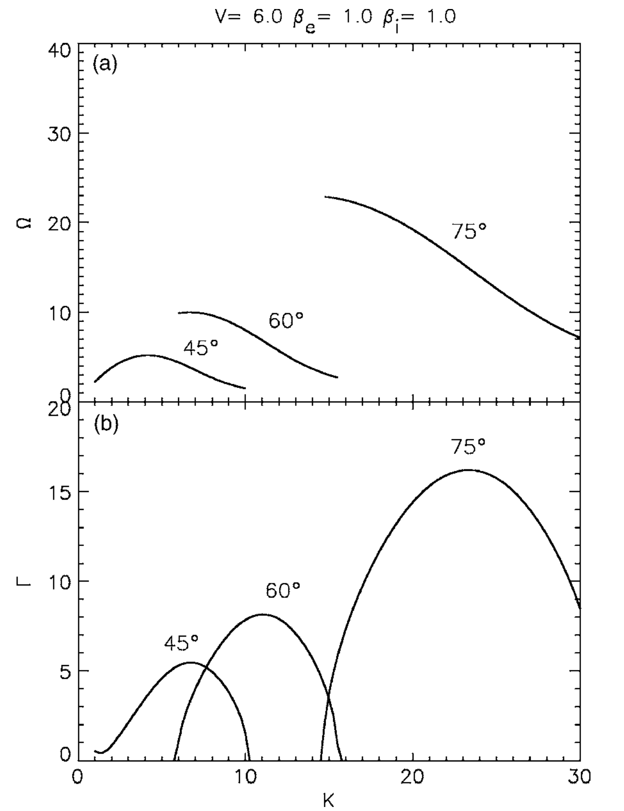

FIG. 5. The normalized frequency $\Omega=\omega/\omega_{ci}$ (a) and growth rate $\Gamma=\gamma/\omega_{ci}$ (b) vs the normalized wave number $K=k\delta_i$.

there is an interaction between the sound wave and this whistler that triggers the instability. This may be because at this point this whistler has negative energy in the ion frame.

For the general quartic let us group the parameters, in one set β_i, β_e, and V and a second set θ and K. For any choice of the first set we get one curve for Ω versus K for each choice of θ. Except near the center of the reconnection layer, β_e and β_i tend to be of order unity. The parameter $V=V_0/V_A$ can be expressed in terms of the layer half thickness δ and the ion skin depth. From $j=neV_0/c\approx B_0/4\pi\delta$, and defining $\delta_i=c/\omega_{pi}$, we get

$$V=\frac{V_0}{V_A}=\frac{\delta_i}{\delta}\frac{B_0}{B}, \tag{17}$$

the ratio of the ion skin depth to the half thickness of the layer times B_0/B, where B is the local value of magnetic field. The latter factor arises from V being defined in terms of the local Alfvén speed. (V is insensitive to the local value of density, so we can take the density as the outside density.) Experimentally it is found that $\delta/\delta_i\approx 0.35$ for nearly all cases, so half way to the center $V\approx 6$.

Let us take $V=6$, $\beta_e=1$, and $\beta_i=1$. Then we plot real part of Ω as a function K in Fig. 5(a) for $\theta=45°$, 60°, and 75°. We plot Γ the imaginary part of Ω in the same way in Fig. 5(b). It is seen that at the larger angle, Γ is quite large and the wave numbers are also very large corresponding to a very small wavelength with a very short life time. This justifies to some extent our local approximation.

The value of Ω refers to the ion frame. In the laboratory frame, the ions could have an appreciable drift velocity $V_D\approx\alpha V_A$ of order the ion thermal speed so that the observed frequency may be Doppler shifted from Ω by $-K\alpha$. For example, if $\alpha=1$ the observed scaled frequency would be $\Omega_{\rm obs}=\Omega-K$. Thus, the observed frequencies could be considerable lower than the ones calculated for the ion frame. They could be anywhere from zero up to the solid lines or even greater. The growth rates Γ are of course unaffected. In fact, there is an indication that, in the laboratory frame, some of the largest intensities are seen at low frequencies.

The fact that small and large K modes are stable and intermediate ones unstable for fixed V and θ is clearly seen in Fig. 5(b). In spite of the crudeness of our treatment, the large growth rates calculated by the theory support the fact that the instability discussed here is actually responsible for the observed fluctuations.

IV. THE WAVE RESISTIVITY

We may use quasilinear theory to calculate the contribution to resistivity due to the waves. We calculate the force in the x direction due to the waves.

There are three contributions of these waves to the force on the electrons. The one we consider first is the $\mathbf{j}_1\times\mathbf{B}_1$ Lorentz force. Let us first show that the Lorentz force on the whole plasma, electron, and ions is zero, when averaged over scales larger than the wave scale:

$$\langle\mathbf{j}\times\mathbf{B}\rangle=\frac{1}{4\pi}\langle(\nabla\times\mathbf{B})\times\mathbf{B}\rangle=\left\langle\nabla\cdot\left(\frac{\mathbf{BB}}{4\pi}-\mathbf{I}\frac{B^2}{8\pi}\right)\right\rangle=0, \tag{18}$$

where $\mathbf{I}$ is the unit dyadic, (since the spatial gradients of physical quantities always average out.)

Thus, the force on the electrons is the opposite of the force on ions, and we calculate the latter force for single mode and take its negative to get the force on the electrons. For cold ions, we have

$$\mathbf{j}_{1i} = \frac{4\pi i n_0 e^2}{M\omega c}\mathbf{E} \tag{19}$$

and

$$\mathbf{B}_1 = \frac{\mathbf{k}c \times \mathbf{E}}{\omega}. \tag{20}$$

This latter $\mathbf{E}$ is the electromagnetic one, and only the E_2 and E_3 components enter into it.

Now consider the average with $\mathbf{j}_1=\frac{1}{2}(\hat{\mathbf{j}}e^{i\phi}+\hat{\mathbf{j}}^*e^{-i\phi})$ and similarly for $\mathbf{B}_1$ where $\phi=\mathbf{k}\cdot\mathbf{r}-\omega t$ and $\hat{\mathbf{j}}$ and $\hat{\mathbf{B}}$ are amplitudes of $\mathbf{j}$ and $\mathbf{B}$. We then have

$$\langle \mathbf{j}_1 \times \mathbf{B}_1\rangle = \frac{1}{4}\hat{\mathbf{j}}_1 \times \hat{\mathbf{B}}_1^* + \text{c.c.}, \tag{21}$$

where the $\hat{\mathbf{j}}_1\times\hat{\mathbf{B}}_1$ and the $\hat{\mathbf{j}}_1^*\times\hat{\mathbf{B}}_1^*$ terms average out and where c.c. denotes complex conjugate as usual. Thus, the x component of the Lorentz force of the wave on the ions is

$$\begin{aligned}\langle \mathbf{j}_1 \times \mathbf{B}_1\rangle_x &= \frac{i}{4}\frac{\omega_{pi}^2}{4\pi\omega}\left(\hat{\mathbf{E}} \times \frac{\mathbf{k}\times\hat{\mathbf{E}}_3^*}{\omega^*}\right)\cdot\hat{\mathbf{x}} + \text{c.c.}\\ &= -\frac{i}{4}\frac{\omega_{pi}^2}{4\pi\omega\omega^*}(\mathbf{k}\cdot\hat{\mathbf{E}})(\hat{\mathbf{E}}_3^*\cdot\hat{\mathbf{x}}) + \text{c.c.}\\ &= -\frac{i}{4}\frac{\omega_{pi}^2}{4\pi\omega\omega^*}k\hat{E}_1(-\hat{E}_3^*\cos\theta) + \text{c.c.},\end{aligned} \tag{22}$$

where $\mathbf{E}_3$ is the electromagnetic E_3 vector.

But from Eq. (11)

$$E_3 = -\cos\theta\frac{\Omega^2 - K^2\beta_e/2}{\Omega^2\sin\theta - KV\Omega}E_1 = -\cos\theta A_{13}E_1, \tag{23}$$

so the x component of the average $\mathbf{j}\times\mathbf{B}$ force on the electrons is the negative of Eq. (21) and can be written as

$$\mathbf{F}_{xe} = k\frac{|\hat{E}_1|^2}{8\pi}\frac{\omega_{pi}^2}{\omega\omega^*}\cos^2\theta\,\text{Im}(A_{13}). \tag{24}$$

This quantity is negative and gives the force in the opposite direction to the electron drift velocity $\mathbf{V}_0$ of the electrons. The forces in the other directions average out by symmetry when summed over the waves.

This force is nonzero only when γ is nonzero, and the mode is growing. It is only produced by the unstable mode. One expects that when the wave reaches a certain amplitude it will saturate and disappear due to nonlinear processes. As the wave disappears, the force on the electrons will probably reverse but the decay behavior is of a different nature and it is unlikely that it will totally cancel the force produced when the wave is growing, so one expects that the above formula should give a reasonable estimate of the mean Lorentz force on the electrons. Notice that the formula is dimensionally correct since it is a dimensionless constant times the gradient of the energy density of the electrostatic field.

The other two contributions to the wave force arise from the x component of the electrostatic force $-n_1eE_x$. In a manner similar to the above calculation of the Lorentz force, we can reduce this to

$$\langle -n_1eE_{x1}\rangle = -\frac{i}{4}\frac{\omega_{pi}^2}{\omega^2}(|\hat{E}_1|^2\sin\theta - \hat{E}_1\hat{E}_3^*\cos\theta) + \text{c.c.}. \tag{25}$$

Note that the two contributions come from the electric vector in the x-z plane, both the E_1 and E_3 vectors contributing. Again expressing E_3 in terms of E_1, we get

$$\begin{aligned}\langle -n_1eE_{x1}\rangle = \omega_{pi}^2 k\frac{|\hat{E}_1|^2}{8\pi}\Bigg[&\text{Im}\left(\frac{1}{\omega^2}\sin\theta\right)\\ &+ \text{Im}\left(\frac{A_{13}}{\omega^{*2}}\right)\cos^2\theta\Bigg].\end{aligned} \tag{26}$$

The first term, due to the electrostatic field, is larger than the second and also than the $\mathbf{j}_1\times\mathbf{B}_1$ force by at least an order of magnitude, so that the force on the electrons is basically due to the $|E_1|^2$ contribution alone, and the other contributions can be ignored.

Again, because of charge neutrality, the total electrostatic force on the plasma is zero. Thus the unstable wave holds no momentum and the force it exerts on the electrons can be thought to react back immediately on the ions, just as happens in electron-ion collisions.

We still must express $|E_1|^2$ in terms of $|\mathbf{B}_1|^2$ in order to use the value of the measured magnetic fluctuation amplitudes to evaluate the wave force. Now,

$$|\mathbf{B}_1|^2 = \frac{k^2c^2}{|\omega^2|}(|E_2|^2 + |E_3|^2) \tag{27}$$

with no direct contribution from $|E_1|^2$. Thus we need to express E_2 in terms of E_1; E_3 is already so expressed. E_2 is found in terms of E_1 from Eq. (10) using the expression for E_3 in terms of E_1. The final result for the total x force in terms of the energy of the magnetic perturbations, $\langle(\delta\mathbf{B})^2\rangle/8\pi=|\hat{\mathbf{B}}_1|^2/16\pi$, can be written as

$$\begin{aligned}F_x = 2\frac{\Omega\Omega^*}{K^2A}\,\text{Im}\Bigg[&\frac{\sin\theta}{\Omega^2} + \cos^2\theta A_{13}\left(\frac{1}{\Omega\Omega^*}\right.\\ &\left.+\frac{1}{\Omega^{*2}}\right)\Bigg]\frac{k\langle(\delta\mathbf{B})^2\rangle}{8\pi},\end{aligned} \tag{28}$$

where A is the constant relating $|E_2|^2+|E_3|^2$ to $|E_1|^2$. That is,

$$A = |A_{13}|^2\cos^2\theta + |A_{12}|^2, \tag{29}$$

where

$$(K^2+1)A_{12} = i\frac{\sin\theta}{\Omega}\left(\Omega^2 - \frac{\beta_e}{2}K^2\right) + i\Omega A_{13}\cos^2\theta. \tag{30}$$

One can finally write

$$F_x = -kC\frac{\langle(\delta\mathbf{B})^2\rangle}{8\pi}, \tag{31}$$

where, as stated above, $\langle(\delta\mathbf{B})^2\rangle/8\pi$ is the average energy of the magnetic fluctuations in a single mode. The total force is

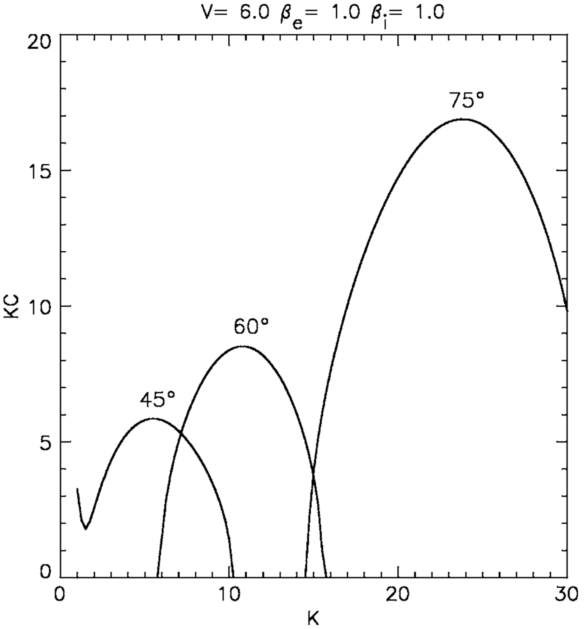

FIG. 6. The parameter $CK=Ck\delta_i$ vs the normalized wave number $K=k\delta_i$.

obtained by summing Eq. (31) over all the unstable modes.

However, it turns out that for most choices of the parameters, C is of order unity, so one can say that the force is essentially the negative gradient of the magnetic perturbation energy.[4]

V. THE WAVE RESISTIVITY

One can now express the force on the electrons as a wave resistivity η_w to investigate the influence of the waves on the reconnection process. If η_w is the resistivity in electromagnetic units, then the force on the electrons is

$$ne\eta_w j = kC\frac{\langle(\delta\mathbf{B})^2\rangle}{8\pi}, \tag{32}$$

where $j=B_0/4\pi\delta$ is in electromagnetics units.

Thus,

$$\eta_w = \frac{kC\langle(\delta\mathbf{B})^2\rangle/8\pi}{neB_0/4\pi\delta}\ \text{emu} \tag{33}$$

or

$$c\eta_w = 3\times 10^6\frac{\delta}{\delta_i}\frac{KC}{n_{13}}\frac{\langle(\delta\mathbf{B})^2\rangle}{B_0}\ \text{cm}^2/\text{s}, \tag{34}$$

where $n=n_{13}10^{13}$, δB and B_0 are in gauss, $\delta_i=c/\omega_{pi}$ is the ion skin depth, and as defined earlier, $K=k\delta_i$ is our dimensionless wave number. This is to be compared with the perpendicular Spitzer resistivity taken at the measured value of the electron temperature, $T_e=6$ eV,

$$c\eta_{\text{Sp}} = 6.4\times 10^6\ \text{cm}^2/\text{s}. \tag{35}$$

The expression for C, although complicated, can easily be evaluated on the computer for any choice of the parameters. In Fig. 6, we give KC as a function of K for the same parameters as we used in Fig. 5, for the same three angles, and for the same $V=6$.

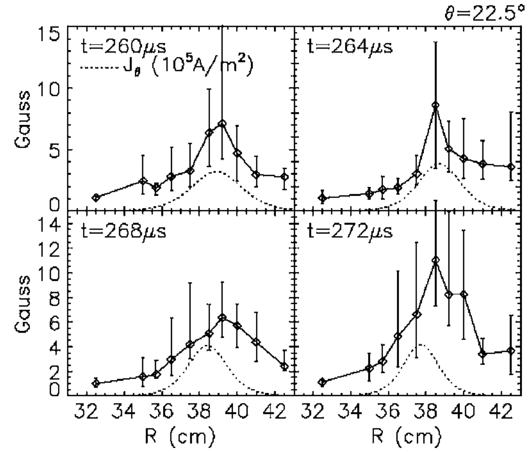

FIG. 7. The magnetic fluctuation intensity above 250 kHz and summed over angles as a function of radius at different times during the reconnection.

The measured values for the root-mean-square amplitude of the magnetic fluctuations, with frequencies above 250 kHz and summed over all angles, are shown in Fig. 7. For our estimate of η_w, we take the measured of δB at about half way from the current layer center to its edge, which is consistent with our assumption of magnetized electrons.

It is not known precisely from which frequencies and propagation angles in the ion rest frame the largest contribution to $\langle(\delta\mathbf{B})^2\rangle$ comes because, as mentioned above, it is difficult to compare the frequencies in the ion frame with those in the laboratory frame. Therefore, we simply choose the value of KC at the largest angle 75° and at the K that has the maximum growth rate. For $V=6$ the value of KC is 17. We take the ratio $\delta/\delta_i\approx 0.35$, the usual ratio,[16] $\langle(\delta\mathbf{B})^2\rangle=(10\ \text{G})^2$ and $B_0=100$ G. This yields

$$c\eta_w = 3\times 10^6\times 0.35\times 17\times\frac{100}{100} = 1.7\times 10^7\ \text{cm}^2/\text{s}, \tag{36}$$

a value about three times the Spitzer value and consistent with the measured ratio η^*/η_{Sp}.

There are many uncertainties in this calculation. K and θ are chosen to give the largest growth rate, and there is no guarantee that they lead to the correct effective frequencies for the main fluctuations. η^* measured at the X point could be dominated by the $\nabla\cdot\mathbf{P}_e$ term, so there is no particular reason to compare η_w with it. However, we see in the following section that if the heating is done by the waves, and if the plasma pressure is to be raised by the waves enough to balance the decreased field pressure then we actually do need $\eta_{\text{Sp}}+\eta_w\approx\eta^*$ in the more general case.

VI. WAVE HEATING

The process by which the waves heat the plasma has not been included in our linear and nonlinear theories. However, the wave force on the electrons $\mathbf{F}_w$ times their velocity $\mathbf{V}_{De}$ relative to some basic velocity frame, in which their total energy is unchanged, produces an amount of heating H_e, where

$$H_e = -\mathbf{F}_w \cdot \mathbf{V}_{De}. \tag{37}$$

This is true because in the basic frame, if there were no reconnection electric field to maintain $\mathbf{V}_{De}$ the electrons would change their kinetic energy at the rate

$$\frac{d}{dt}\frac{1}{2}(\rho_e V_{De}^2) = \rho_e \mathbf{V}_{De} \cdot \frac{d\mathbf{V}_{De}}{dt} = \mathbf{F}_w \cdot \mathbf{V}_{De} \tag{38}$$

and since their total energy is fixed in this frame, this loss of energy must go to heating the electrons at the rate H_e given by Eq. (37).

Similarly, the ions are heated at the rate

$$H_i = -\mathbf{F}_i \cdot \mathbf{V}_{Di}, \tag{39}$$

so the total heating of the electrons plus ions is

$$H_i + H_e = \mathbf{F}_e \cdot (\mathbf{V}_{Di} - \mathbf{V}_{De}) = -\mathbf{F}_e \cdot \mathbf{V}_0, \tag{40}$$

where $\mathbf{V}_0$ is the electron-ion relative velocity. Now, the force on the electrons on the electrons is

$$F_e = -(\eta_{\mathrm{Sp}} + \eta_w) nej, \tag{41}$$

so the total heating rate of the plasma is

$$(\eta_{\mathrm{Sp}} + \eta_w) cjneV_0 = (n_{\mathrm{Sp}} + \eta_w) cj^2 \tag{42}$$

in complete analogy to the Ohmic heating rate by electron-ion collisions.

Now let us compare this with the heating required to bring the plasma pressure into balance with the reduced magnetic pressure during reconnection.

The incoming plasma has an $\mathbf{E} \times \mathbf{B}$ velocity of

$$v_R = \frac{cE_r}{B} = \frac{c\eta^* j}{B}, \tag{43}$$

where E_r is the reconnecting electric field expressed in term of η^*, and it must be heated in a time of order

$$t_R = \frac{\delta}{v_R} = \frac{\delta B}{\eta^* jc} \tag{44}$$

and the plasma pressure has to be raised by $\approx B^2/8\pi$ in this time, so the heating rate must be of order

$$H \approx \frac{B^2}{8\pi t_R} = \frac{B^2}{8\pi}\frac{\eta^* jc}{\delta B} \approx \frac{1}{2}\eta^* cj^2, \tag{45}$$

so if the heating is produced by anomalous resistivity then we need $\eta^* \approx \eta_{\mathrm{Sp}} + \eta_w$.

Also to avoid the B/n constraint that decreases n as B decreases, we need the plasma to slip a distance δ in the reconnection time or, from Eq. (2), we need

$$v_\eta = j(\eta_{\mathrm{Sp}} + \eta_w)c/B \approx j\eta^* c/B, \tag{46}$$

so again we need $\eta_{\mathrm{Sp}} + \eta_w \approx \eta^*$.

VII. CONCLUSION

Significant magnetic fluctuations are observed in the MRX during experiments in which the reconnection rate is greater than the classical Sweet-Parker rate. In this paper, we show that these waves can produce a wave resistivity sufficiently larger than the perpendicular Spitzer resistivity to produce the enhanced reconnection rate. We assume that these waves are generated by a recently discussed instability. We base our calculation on a simplified linear theory of the instability and derive the effective resistivity from the quasilinear extension of the theory. The resistive force produced by the waves is a constant of order unity times the negative gradient of the magnetic fluctuation energy in the waves.

The actual calculation of the wave resistivity depends on the plasma having a large perpendicular current density, and is independent of particular choice of the reconnection model. We show that even if there were no binary collisions or collisions related to wave resistivity, the plasma is able to break its magnetic lines of force. However, some sort of collisions are needed to allow the electrons to slip across the lines far enough to break the approximate (collisionless) constraint of B/n being a fluid constant of the motion. Further, collisions are needed to raise the pressure of the plasma by $B^2/8\pi$ as it enters the current layer. (This increase is necessary to balance the outside total pressure by the reduced magnetic pressure and this enhanced plasma pressure.) For the experiment in which the fluctuations are observed, ordinary binary collisions are inadequate for these two purposes. However, when the resistivity due to waves is added to these collisions, the combined effect is able to accomplish these two goals: the necessary slippage of the plasma to the field lines and its pressure rise as it moves into the current layer. The interaction of fluctuations with other processes in collisionless plasmas, such as those related to the production of quadrupole magnetic fields[17] out of the plane of reconnection, will be considered in the future work.

Our conclusion is that in these MRX magnetic reconnection experiments, magnetic fluctuations play a significant role.

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